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Evolutionary ACO algorithms for truss optimization problems

Buntara S. Gan[a,*], Takahiro Hara[a], Aylie Han[b], Sofia W. Alisjahbana[c], Sholihin As'ad[d]

[a]*Department of Architecture, College of Engineering, Nihon University, Koriyama, Japan*
[b]*Civil Engineering Department, Diponegoro University, Semarang, Indonesia*
[c]*Civil Engineering Department, Bakrie University, Jakarta, Indonesia*
[d]*Civil Engineering Department, Universitas Sebelas Maret, Surakarta, Indonesia*

Abstract

Over the last decade, researchers have proposed several ant colony optimisation algorithms to solve combinatorial problems. Ant Colony Optimisation (ACO) was introduced by Dorigo et al. in the early 1990s and is based on the behaviour of natural ant colonies, in particular the foraging behaviour of real ant species. The indirect communication of real ants in the colony uses pheromone trail lying on the path to find the shortest trail between their food source and the nest. Recently, Evolutionary ACO algorithms have been proposed to solve truss optimisation problems (EACO algorithms). This algorithm can solve truss size and topology problems, which makes EACO very attractive to solve non-combinatorial optimisation problems. Computational tests are described to show the effectiveness of the EACO.

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Keywords: Ant Colony Optimisation; Evolutionary Algorithm; Truss Structure; Topology Optimisation; Size Optimisation.

1. Introduction

Optimal structure design is a very interesting and important topic in the field of engineering and construction. Solving structural optimisation problems is usually defined by a process of finding an optimal design of the structure with minimum material, subjected to some design constraints and specific loading conditions. The optimal design methodology of structures can be categorised by sizing, shape and topology optimisations [1]. Sizing optimisation is a process to optimise a set of sizing structural parameter such as length; thickness; and area, in a predetermined

* Corresponding author. Tel.: +81-24-956-8735.
E-mail address: buntara@arch.ce.nihon-u.ac.jp

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geometry of a structure. Shape optimisation is intended to obtain an optimal shape of the structure by changing the geometrical configuration without changing the sizing and material parameters. In topology optimisation, an initial structural configuration is made that satisfies a set of design criteria without predefined geometrical configuration of the structure. Thus, by optimising the shape and size parameters, a new and innovative structural geometry is sought. Recently, several bio-inspired mechanisms in nature have gained popularity in solving complex combinatorial optimisation problems of structures [2–5].

Structural optimisation problems can be divided into a continuum and discrete design variable. Depending on the type of structural element modelling (viz. beam, plate, block), the most appropriate optimisation methodology is usually applied. There have been many studies on the optimisation of a truss structure [2,4,5]. Most of the optimisation studies on truss problems are based on the discrete design variables assumption, where each member of the truss structure is treated as having separate design variables (i.e. length, thickness, cross-sectional area). The drawback associated with truss optimisation problems is the determination of objective and penalty functions, which is necessary to estimate the feasible solution and to penalise the infeasible solution that is initially unknown.

Ant colony optimisation (ACO) [6] was originally developed for solving combinatorial problems such as the well-known travelling salesman problem. Combinatorial problems are challenging research topics since solving them may involve the utilisation of a vast number (most of them exponentially) of combinations, which are known as the combinatorial explosion phenomenon when the problem to be solved is increased. Kaveh and Talatahari [7] conducted a study of truss optimisation using the particle swarm ant colony optimisation method, which is based on the modified feasible based mechanism.

In the continuum design variable, an evolutionary algorithm inspired by the principle of biological evolution is often used for solving structural topology optimisation problems [8]. The bi-directional evolutionary structural optimization (BESO) [9] algorithm improved its precursor, the evolutionary structural optimization (ESO) [10], by allowing an element in the structure domain to be removed and added simultaneously.

This study introduces the evolutionary ant colony optimisation (EACO) formulation to treat a truss structure as a topology continuum optimisation problem by combining both ACO and BESO algorithms. In the original ACO algorithm, the ant best trail finding process is ignited merely by the roulette-based probability that depends on the amount of pheromones laid on the paths connecting one city. This phenomenon will create a stagnant optimisation solution where all ants end up doing the same tour. To remove the drawback in ACO and to improve the pathfinding process, the pheromone on the paths of the trail will be evolved following BESO by adopting the so-called sensitivity number evaluation.

2. Size and topology optimisation of truss structure using EACO

The objective of EACO optimisation of the truss structure is the minimisation of compliance energy subject to the predefined total volume of the truss. The problems of size and topology EACO optimisation of the truss structure is formulated as follows:

$$\begin{aligned} &\text{minimize} \quad \mathrm{C}(\mathbf{A}) = \mathbf{d}^{\mathrm{T}}\, \mathbf{K}\, \mathbf{d} \\ &\text{subject to} \;\; \sum_{e=1}^{numel} A_e\, \ell_e \le V^*, \end{aligned} \tag{1}$$

where $\mathrm{C}(.), \mathbf{d}, \mathbf{K}$ represent the total compliance energy, displacement vector, and stiffness matrix, respectively. A_e, ℓ_e, V^* denote the cross-sectional area, length of the element e and total volume constrained, respectively.

3. Evolutionary Ant Colony Optimisation (EACO)

In EACO, a node of the truss structure is reflected as a city, while the cross-sectional area of a member in the truss structure is imitated by the pheromone on the path (i.e. a member of the truss structure) in the travelling salesmen problem. A trail is created as a list of the path that an ant travels across to all the cities. The capability of

BESO has proven successful in allowing an element in the structure domain to be removed and added simultaneously to replace the roulette-based probability pathfinding scheme in ACO.

3.1. BESO sensitivity number

In EACO, the selection of the best path and pheromone updating process is decided based on the so-called sensitivity number evaluation, which is adopted from BESO. The sensitivity number of the truss structure is computed from the member strain energy density as

$$\alpha_i^e = \frac{\left(\frac{1}{2}\mathbf{d}_i^T\,\mathbf{K}_i\,\mathbf{d}_i\right)}{A_i\,\ell_i}, \tag{2}$$

where $\mathbf{d}_i$ is the nodal displacement vector of the *i*-th member, and $\mathbf{K}_i$ is the member stiffness matrix. Below is a procedure for updating the pheromone (cross-sectional area) on the member:

- The sensitivity number of each member calculated using Eq. (2) is summed up in descending order from the largest value of pheromone from the previous simulation;
- During the summation process, when the accumulated volume reaches the volume constraint at that time, the sensitivity number is prescribed as the threshold boundary α_{th} between adding and removing the pheromones in all members of the truss structure.

3.2. ACO pathfinding

In ACO, the pathfinding is based on the probability of an ant at a current city going to another city connected by a path is defined as

$$\mathrm{Pr}_{ij} = \frac{\tau_{ij}}{\sum_{k=1}^{ijnode}\tau_{ik}}, \tag{3}$$

where $\mathrm{Pr}_{ij}, \tau_{ij}$ are a probability value for finding a path from *i*-node to *j*-node and the decision index value between *i*-node and *j*-node, respectively. The decision index value is defined as

$$\tau_{ij} = \begin{cases} \left|\log\left|\alpha_{ij} - \alpha_{th} + 1\right|\right| & if\,\alpha_{ij} \geq \alpha_{th} \\ \left|\log\left|\alpha_{th} - \alpha_{ij} + 1\right|\right| & if\,\alpha_{ij} < \alpha_{th} \end{cases}. \tag{4}$$

3.3. EACO pheromone updating

The pheromone updating rule in the EACO follows the original concept of ACO, where the pheromones on the paths in the trail that an ant visits will be updated. On the contrary, pheromones on the paths that the ant does not visit will be evaporated following the BESO removal rule.

3.3.1. Pheromone updating rule

The pheromones on the paths in a trail are updated according to the formula:

$$Ph_e^t = Ph_e^{t-1} + \Delta Ph_e^t, \tag{5}$$

where $Ph_e^t, Ph_e^{t-1}, \Delta Ph_e^t$ denote the current, previous and increment of pheromone (cross-sectional area) intensity on the path (member *e* connected by nodes *i* and *j*) at simulation time *t*, respectively.

3.3.2. Incremental pheromone

The pheromones on the paths in a trail are updated according to the formula:

$$\Delta Ph_e^t = \begin{cases} +Q \times A_{\max} & if\ i, j \in Trail^t \cap \alpha_{ij}^t \geq \alpha_{th}^t \\ +Q \times A_{\max} & if\ i, j \in Trail^t \cap \alpha_{ij}^t < \alpha_{th}^t \\ -\rho \times A_{\max} & if\ i, j \notin Trail^t \end{cases}, \tag{6}$$

where $Q, \rho, A_{\max}$ are the constant increment rate, constant evaporation rate, and prescribed maximum cross-sectional area, respectively.

3.3.3. Normalisation

In EACO, the evolutionary process of increasing and decreasing the amount of pheromones at every simulation step is defined as

$$\Delta V_e^t = \frac{\Delta Ph_e^t \times \ell_e}{\sum_{e=1}^{numel} \left| \Delta Ph_e^t \times \ell_e \right|} \times ER \times V_e^{t-1}, \tag{7}$$

where $\Delta V_e^t, V_e^{t-1}, ER$ are the increment of the member's volume, previous stored total volume of the truss structure and the evolutionary ratio, respectively.

4. Numerical examples

Two truss structures are optimised using EACO. Size and topology optimisation are considered. The final results are compared to the solutions of other methods to demonstrate the efficiency of the EACO. For the first example where the constraint of optimisation is other than the total volume, Eq. (1) is replaced by the following formula:

$$\begin{aligned} &\text{minimize} \quad \mathrm{C}(\mathbf{A}) = \mathbf{d}^{\mathrm{T}}\, \mathbf{K}\, \mathbf{d} \\ &\text{subject to} \quad \sigma_e \leq \sigma_{\max} \\ &\qquad\qquad\quad \delta_e \leq \delta_{\max} \end{aligned} \quad . \tag{8}$$

where $\sigma_e, \sigma_{\max}, \delta_e, \delta_{\max}$ are the stress of member *e* and its maximum design value, and the displacement and its maximum design value, respectively.

4.1. Size optimisation of 15-bar planar truss structure

A 15-bar planar truss structure, shown in Fig. 1, has previously been analysed [5,11] using particle swarm and genetic algorithm optimisations. The material density is 7800 kg/m^3 and the modulus of elasticity is 200 GPa. The stresses of the members are subjected to the stress constraint of ±120 MPa. All nodal displacements in both *x* and *y* directions are limited by ±10 mm. Maximum and minimum cross-sectional areas are 750 mm^2 and 113.2 mm^2. In EACO, five ants are used to find the optimal designs. The values of $Q = 0.10, \rho = 0.10, ER = 1\%$ are used in the simulation. The design variable is the cross-sectional areas of all the members. The loading conditions applied to the truss structure are P=35 kN, P_2=35 kN, and P_3 = 35 kN, respectively.

Table 1 shows a comparison of present optimal design results with other works. Fig. 2 depicts the convergence graphs of total weight and compliance energy of EACO.

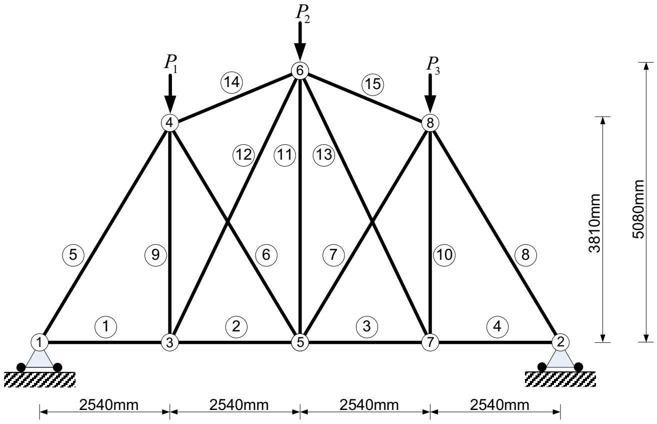


Fig. 1. A 15-bar planar truss structure.

Table 1. Comparison of optimal designs for the 15-bar planar truss structure

Cross-sectional area (mm^2)	Zhang [11]	HPSO [5]	EACO
A_1	308.6	113.2	113.2
A2	174.9	113.2	113.2
A3	338.2	113.2	113.2
A4	143.2	113.2	113.2
A5	736.7	736.7	736.7
A6	185.9	113.2	113.2
A7	265.9	113.2	113.2
A8	507.6	736.7	736.7
A9	143.2	113.2	113.2
A10	507.6	113.2	113.2
A11	279.1	113.2	113.2
A12	174.9	113.2	113.2
A13	297.1	113.2	113.2
A14	235.9	334.3	334.3
A15	265.9	334.3	334.3
Weight (kg)	142.117	105.735	105.735

4.2. Size and topology optimisations of 74-bar planar ground truss structure

This example considers a cantilever-like four-span-two-storey 74-bar planar truss structure: the so-called ground structure [12], as shown in [. 3 along with its loading condition. This structure has previously been analysed [13] using the CONLIN method of optimisation. The modulus of elasticity is 1.0 GPa. Maximum and minimum cross-sectional areas are respectively 500 mm^2 and 0.001 mm^2. In EACO, five ants are used to find the optimal designs. The value of $Q = 0.10, \rho = 0.01, ER = 1\%$ are used in the simulation. The total volume design is constrained at $V^* = 1.0 \times 10^6 \,\text{mm}^3$.

Fig. 4 shows a comparison of present optimal design results with other works [13]. Fig. 5 depicts the convergence graphs of total weight and compliance energy of EACO.

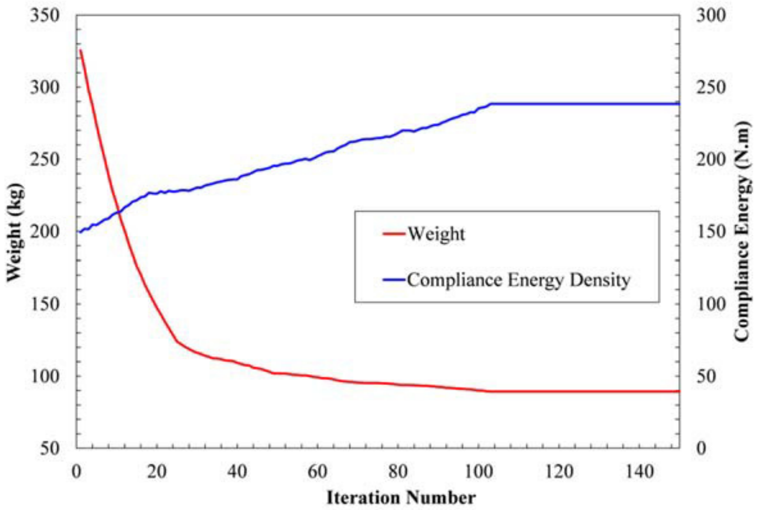


Fig. 2. Convergence history for the 15-bar planar truss structure.

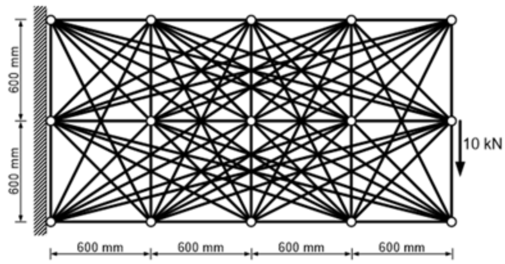
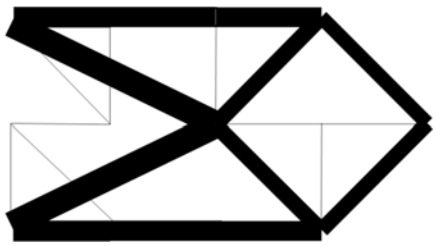
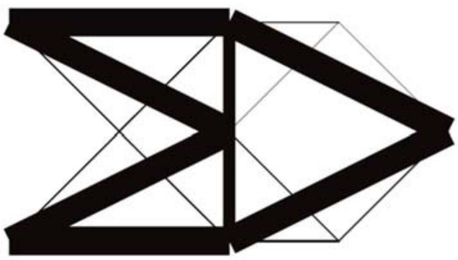


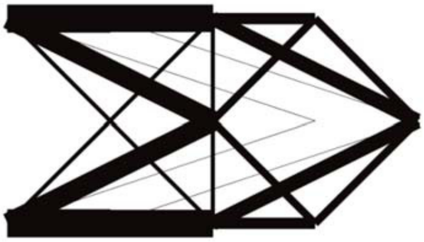
Fig. 3. A 74-bar planar truss ground structure.



(a) EACO, C=80.7 kN.mm



(b) (b) SLP, C=81.0 kN.mm



(c) CONLIN, C=81.5 kN.mm

Fig. 4. Comparison of optimal design between (a) EACO; (b) SLP; and (c) CONLIN methods [13].

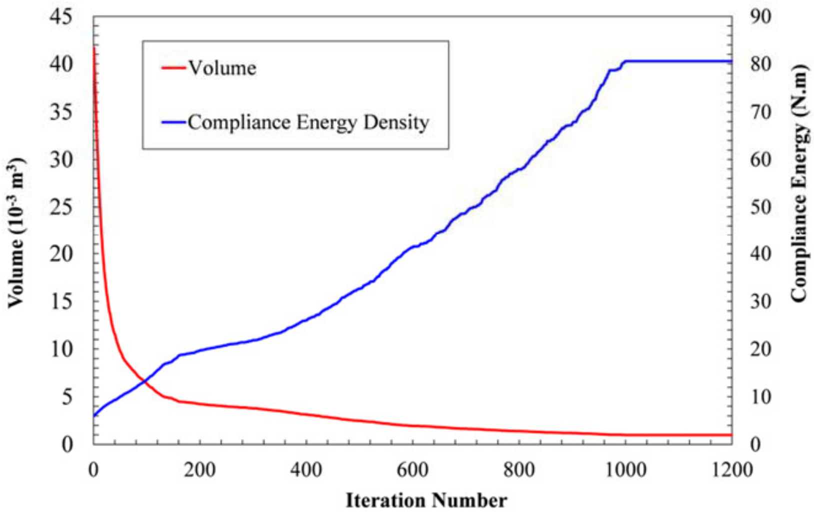


Fig. 5. Convergence history for the 74-bar planar truss ground structure.

5. Results and summary

In Fig. 2, the EACO algorithm converges to the optimal solution after 100 iterations and shows a fast convergence rate, especially during the early iterations. From Table 1, the weight of Zhang [11] had a larger value compared to HPSO and EACO; thus, it was not yet optimal.

Fig. 4 shows a comparison of results between the EACO, SLP and CONLIN methods. It seems that there are many optimal design solutions that meet the objective under the same constraints. As shown in the figure, the EACO achieved the smallest value of compliance energy. Fig. 5 shows the stable convergence rate of the EACO.

The EACO algorithm presented here has been verified by two types of truss structures optimisation problems. All the results show that the EACO algorithm has a stable convergence rate during the iteration, without experiencing any stagnancy solution (i.e. stepping curves) that appeared in the other methods.

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