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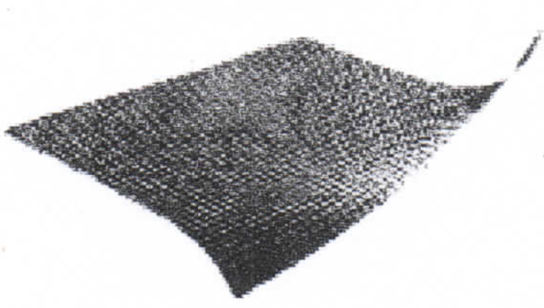

DYNAMICS OF ORTHOTROPIC PLATES WITH MIXED BOUNDARY CONDITIONS SUBJECTED TO MOVING DYNAMIC LOADS WITH MULTIPLE SUDDEN CHANGE OF LATERAL POSITIONS

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Abstract

In this paper the dynamic response analysis of orthotropic rectangular plates subjected to moving dynamic loads with multiple sudden changes of lateral positions is investigated. The orthotropic plate has mixed boundary conditions that is widely faced in engineering applications, but has not been widely addressed to in the literature, partially due to the numerical difficulties. The orthotropic plate is supported by a Pasternak foundation. This type of elastic foundation model is introduced to accommodate shear interactions between the spring elements. The plate's natural frequencies are presented in a form analogous to those of a simply supported plate as wave numbers. These wave numbers are determined from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method. The multiple sudden lateral position change of the moving dynamic load is expressed by the Heaviside generalized function. The homogeneous solution of the problem can be solved by a method of separation of variables. The procedure comprises the derivation of a sequence of solutions of separable form, in such a way that superposition yields a solution satisfying the boundary conditions. The dynamic response of the plate is expressed in integral form that is readily to be solved by using the Duhamel integration method. A numerical example is given, demonstrating the applicability of the theory to orthotropic plates with mixed boundary conditions under a multiple sudden lateral position change of a moving dynamic load.

Keywords: orthotropic plate, mixed boundary condition, Pasternak foundation, Modified Bolotin Method, Duhamel integration method.

1. Introduction

The investigation of dynamic response of orthotropic plates supported by an elastic foundation under dynamic moving loads has been a topic of interest for well over a century for the design of rigid runway and roadway pavements. Such structures are often subjected to dynamic loads with multiple sudden changes of lateral positions. The design of rigid pavements is traditionally based on the analytical solution of an infinitely long plate under an equivalent static load. Such design methods have deficiencies as the dimensions of such structures are finite and the moving vehicles exert dynamic loads of various amplitudes due to the mechanical vibration of engines. The behavior of structures under dynamic loads is different from that under static loads. The load amplitude of the dynamic load is often assumed to be constant. However, the dynamic loads created for example by vehicles in fact have variations in load amplitude with time, resulted from the pavement surface roughness and the mechanical systems of the vehicle. Most of the studies were conducted fo dynamic loads of constant amplitude. Dynamic response of plates resting on an elastic foundation has attracted much less attention, in comparison with the dynamic loads on rigid pavements. The limited investigations involved analytical procedures for plates with simple and regular boundary conditions Gbadeyan and Oni (1992) gave a closed form solution by using a double Fourier sine integra transformation to analyze a simply supported rectangular plate resting on an elastic Pasternak foundation subjected to an arbitrary number of moving concentrated masses. Static and free vibration analyses of plates resting on an elastic foundation had been studied extensively, for example by Saha (1997) and Pevzner et al. (2000). Extensive studies of the dynamic response of plates supported by an elastic foundation with simply supported and unsymmetrical boundary conditions had been investigated by Alisjahbana and Wangsadinata (2005, 2006) using the Modified Bolotin Method (MBM).

In the present research work the problem of an orthotropic rectangular plate on a Pasternak foundation under a multiple sudden lateral position change of a moving load is further studied, whereby the plate is under a very general restraint condition along its supports. The vibration modes are solved using the Modified Bolotin Method. As the mode shapes are expressed as a product of eigenfunctions, the dynamic solution of the plate is obtained on the basis of orthogonality properties of eigenfunctions. The dynamic response of the plate is expressed in integral form, so that it can be readily integrated to obtain the various dynamic responses of the plate.

The geometry and material properties are assumed to be linear elastic and the orthotropic plate in consideration is of finite dimensions. Finally results for dynamic responses such as deflection, bending and twisting moments, shear forces of the plate are presented incorporating the effects of damping ratio and load frequency.

2. Governing Equation

Using the classical theory of thin plates, the equation of equilibrium of an elastic orthotropic plate resting on a Pasternak foundation is as follows:

$$D_x\frac{\partial^4 w}{\partial x^4}+2B\frac{\partial^4 w}{\partial x^2\partial y^2}+D_y\frac{\partial^4 w}{\partial y^4}+\gamma h\frac{\partial w}{\partial t}+\rho h\frac{\partial^2 w}{\partial t^2}+k_f w-G_s\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=p(x,y,t)$$

(1)

where D_x, D_y= plate stiffness in x and y direction; B= plate torsional stiffness; γ= damping ratio; h= plate thickness; ρ= plate mass density; k_f= stiffness of the foundation; G_s= shear modulus of the foundation; t= time; x, y= rectangular Cartesian coordinates in the plane of the plate; p(x,y,t)= dynamic load acting on the plate. The harmonic concentrated load which suddenly moves to a new position at $t=t_1$ and continues to moves again to another new position at $t=t_2$ can be expressed as:

For $0\le t\le t_1$: $p(x,y,t)=P_0(1+\alpha\cos\omega t)\delta\left[x-\left(x_0+\Delta x_1 H(t-t_1)\right)\right]\delta[y-y_0]$

(2)

For $t_1\le t\le t_2$: $p(x,y,t)=P_0(1+\alpha\cos\omega t)\delta\left[x-\left(x_1+\Delta x_2 H(t-t_2)\right)\right]\delta[y-y_0]$

(3)

where P_0= amplitude of the load; α= load coefficient=0.5; ω= angular frequency of the load; x_0= initial position of the load in x direction; Δx_1= sudden position change of the load at $t=t_1$;. Δx_2 sudden position change of the load at $t=t_2$;.and H= Heaviside unit step generalized function.

Consider the following types of support conditions for the plate edge:

along x=0

$$-D_x\left(\frac{\partial^2 w}{\partial x^2}+\nu_y\frac{\partial^2 w}{\partial y^2}\right)=k_1\frac{\partial w}{\partial x}\quad ;w=0$$

(4)

along x=a

$$-D_x\left(\frac{\partial^2 w}{\partial x^2}+v_y\frac{\partial^2 w}{\partial y^2}\right)=k_2\frac{\partial w}{\partial x}\quad ;w=0$$

(5)
along y=0 and y=b

$$-D_y\left(\frac{\partial^2 w}{\partial y^2}+v_x\frac{\partial^2 w}{\partial x^2}\right)=0\quad ;w=0$$

(6)

where υ_x is the Poisson's ratio in x direction, υ_y is the Poisson's ratio in y direction, k_1 is an elastic rotational restraint constant along x=0; k_2 is an elastic rotational restraint along x=a. A model of an orthotropic damped plate with unsymmetrical restraints along its edges resting on an elastic foundation subjected to a moving dynamic load with multiple sudden change in lateral positions can be established.

3. General Analysis

In order to solve the problem described above, it is assumed that the principal elastic axes of the material are parallel to the plate edges and the free vibration solution of the problem is set as:

$$w(x,y,t)=W(x,y)\sin\omega t$$

(7)

where ω is the circular frequency and W(x,y) is a function of the position coordinates only. Then substituting Eq.(7) into the undamped free vibration form of Eq.(1) yields:

$$D_x\frac{\partial^4 W}{\partial x^4}+2B\frac{\partial^4 W}{\partial x^2\partial y^2}+D_y\frac{\partial^4 W}{\partial y^4}-\rho h\omega^2 W+k_f W-G_s\left[\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right]=0$$

(8)

The next step is to find the solution of Eq.(8) with the boundary conditions according to Eq (4), Eq.(5) and Eq.(6), to obtain the eigen frequencies and the mode shapes of the orthotropic plate with mixed support conditions at its edges. By postulating the following eigen frequency, which is analogous to the case of a plate simply supported at all edges (Pevzner et al. 2000), natural frequencies of the system can be expressed as:

$$\omega_{mn}{}^2=\left(\frac{\pi^4}{\rho h}\right)\left[D_x\left(\frac{p}{a}\right)^4+2B\left(\frac{pq}{ab}\right)^2+D_y\left(\frac{q}{b}\right)^4\right]+\frac{k_f}{\rho h}+\frac{G_s}{\rho h}\left[\left(\frac{p\pi}{a}\right)^2+\left(\frac{q\pi}{b}\right)^2\right]$$

(9)

vhere p and q are real numbers to be solved from a system of two transcendental equations, btained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin lethod (Pevzner et al. 2000).

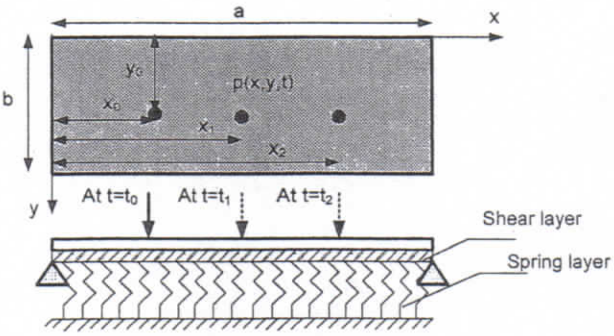

Figure 1. Rectangular orthotropic plate on a Pasternak foundation subjected to dynamic moving loads with multiple sudden changes of lateral positions.

4. Dynamic Response of the Plate

The dynamic response of the plate can be found by using the method of variation of parameters as a general method of determining a particular solution of the corresponding non-homogeneous partial differential equation, which can be written in the following form:

$$w_{mn}(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}X_m(x)Y_n(y)T_{mn}(t) \tag{10}$$

where $X_m(x)$, $Y_n(y)$ are eigenfunctions, $T_{mn}(t)$ is a function of time which must be determined through further analysis.

The differential equation for the coefficient functions $T_{mn}(t)$ can be expressed as:

$$\ddot{T}_{mn}(t)+2\gamma\omega_{mn}\dot{T}_{mn}(t)+\omega_{mn}^2T_{mn}(t)=\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\,\frac{p(x,y,t)}{\rho hQ_{mn}} \tag{11}$$

where Q_{mn} is a normalization factor.

The particular solution of the temporal function $T_{mn}(t)$ can be represented in a form of the Duhamel convolution integral as follows:

$$T^{*}_{mn}(t)=\int_0^t\left[\frac{p(x,y,\tau)}{\rho hQ_{mn}}\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\right]\left[\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}}\sin\left(\omega_{mn}\sqrt{(1-\gamma^2)}\,(t-\tau)\right)\right]d\tau \tag{12}$$

The general solution for the forced response deflection of the plate to an arbitrary dynamic moving load $p(x,y,t)$ is given in integral form as follows:

$$w(x,y,t)=\sum_{m=1}^{m=\infty}X_m(x)\sum_{n=1}^{n=\infty}Y_n(y)\int_0^t\left[\frac{p(x,y,\tau)}{\rho hQ_{mn}}\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\right]\left[\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}}\sin\left(\omega_{mn}\sqrt{(1-\gamma^2)}\,(t-\tau)\right)\right]d\tau \tag{13}$$

Bending moments and vertical shear forces in the plate can be computed in terms of the deflection and its derivatives obtained from Eq. (13).

5. Results

An orthotropic rectangular plate resting on an elastic Pasternak foundation subjected to a dynamic load with multiple sudden change of lateral position is considered. The data for the plate and load amplitude for the numerical examples treated in this section are as follows: a=7.5 m, b=15 m (Figure 1), h=0.5 m, $E_x=3.0\times10^{10}$ N/m^2, $E_y=2.0\times10^{10}$ N/m^2, $G_p=1.09\times10^{10}$ N/m^2, $\upsilon_x=0.15$, $\upsilon_y=0.1$, and $P_0=2\times10^5$ N. The boundary conditions are: partially fixed along the longitudinal edges (x=0, 7.5 m) and simply supported along the shorter edges (y=0, 15m).

In this numerical example, the Pasternak elastic foundation stiffnesses are given as $k_f=7.5\times10^7$ N/m^3 and $G_s=1.0\times10^7$ N/m^3. The load initial position is at (x_0=2.5m and y_0=7.5m) and at $t=t_1$, the load suddenly moves to a new position at (x_1=3.0m and y_0=7.5m). At $t=t_2$, the load suddenly moves to another new position again at (x_2=3.5m and y_0=7.5m).

Due to the mechanical systems of the vehicle, the moving loads exerted by the vehicle might not have a constant amplitude, and a moving harmonic load model is generally used in practical analysis (Kim & Roesset 1998). In this paper a single moving harmonic concentrated load $P_0(1+\alpha\cos\omega t)$, suddenly moving from one position to another position along the middle line (y_0=7.5m) is considered.

Figure 2 shows the response spectra under the moving harmonic load when foundation stiffness is given as k_f=7.5x10^7 N/m^3 for various damping ratios. It can be seen that the damping ratios play an important factor to reduce the dynamic deflection of the system.

Figure 3 illustrates the effect of the magnitude of the sudden x position change on the maximum deflection under the moving load for various damping ratios. As expected, the maximum dynamic deflection increases as the change in x position increases for all values of damping ratio.

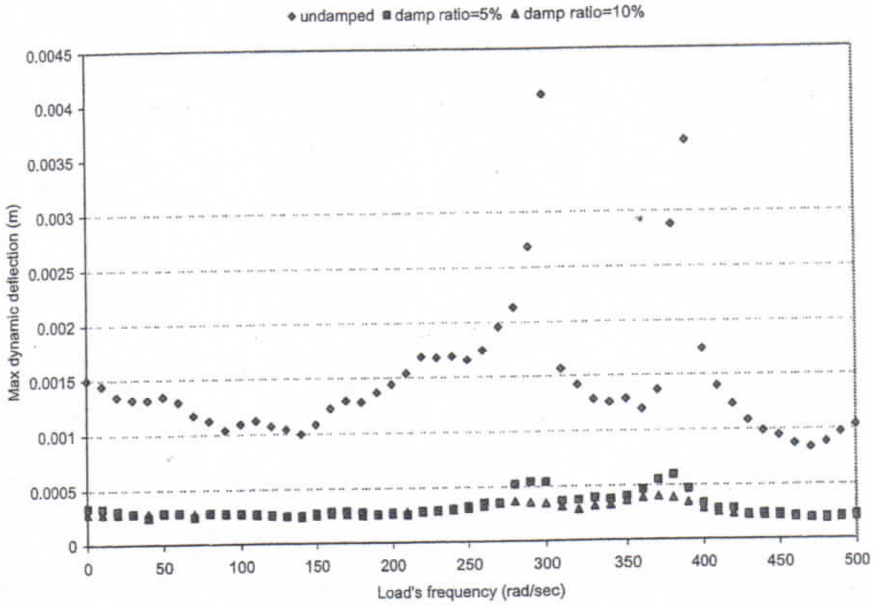

Figure 2. Response spectra as a function of load's frequency for various values of damping ratios.

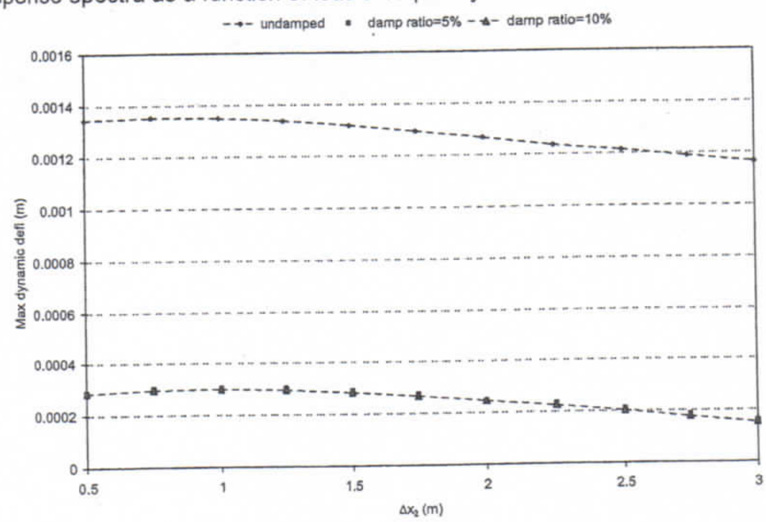

Figure 3. Response spectra as a function of the magnitude of the sudden x position change (Δx_2) for various values of damping ratios.

The bending moments and shear forces change with the increase of the damping ratio for the value of ω=500 rad/s which is far away from the first natural frequency of the system (Figure 4-Figure 9).

Figure 10 shows the response spectra as a function of load's frequency for various values of elastic foundation stiffnesses. It is obvious that when the plate rests on a weak foundation, the dynamic deflection becomes smaller as the foundation stiffness increases. Figure 11 and Figure 12 show the time history at central point for γ=0% and ω=50 rad/s. It can be seen that by increasing the foundation stiffness by 10 times, the maximum value of dynamic deflection decreases drastically.

The effect of the sudden change of position of the dynamic load on the response is investigated next. Figure 13 shows the shape of the total dynamic deflection at $t=t_0$, when the load suddenly moves to its new position. It can be seen that the deflected area is concentrated only under the dynamic load. Finally, Figure 14 shows the total dynamic load deflection shape at $t=t_1$, when the load suddenly moves to another new position. It can be seen that the deflected area almost covers the entire plate due to the affect of the first load's configuration.

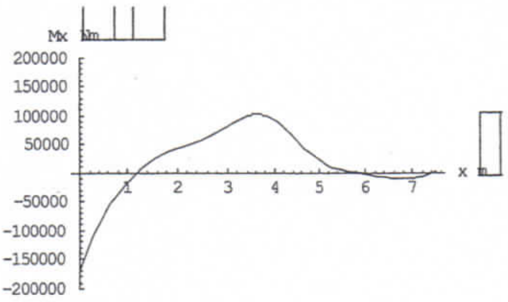

Figure 4. M_x distribution along the x-axis for γ=0%, ω=500 rad/s computed at t=5 s.

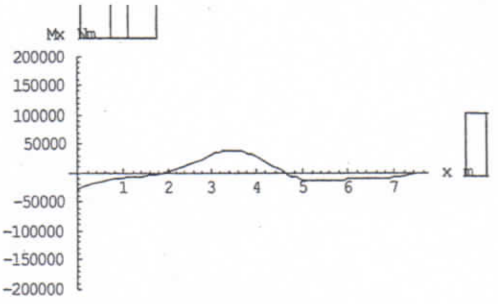

Figure 5. M_x distribution along the x-axis for γ=10%, ω=500 rad/s computed at t=5 s.

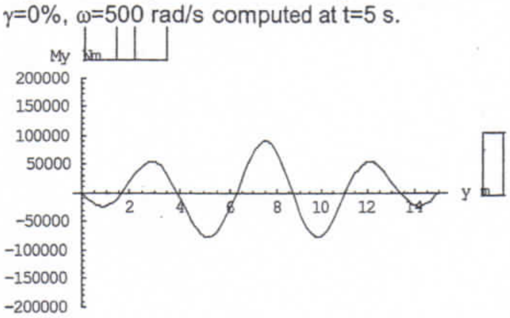

Figure 6. M_y distribution along the x-axis for γ=0%, ω=500 rad/sec computed at t=5 s.

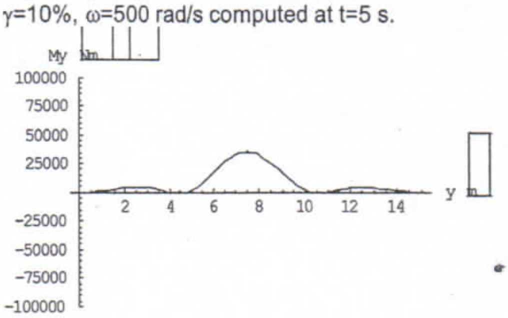

Figure 7. M_y distribution along the x-axis for γ=10%, ω=500 rad/s computed at t=5 s.

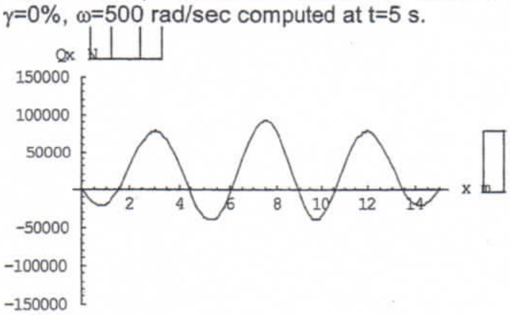

Figure 8. Q_x distribution along the x-axis for γ=0%, ω=500 rad/ computed at t=5 s.

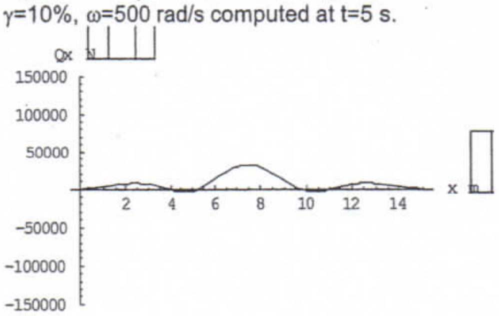

Figure 9. Q_x distribution along the x-axis for γ=10%, ω=500 rad/ computed at t=5 s.

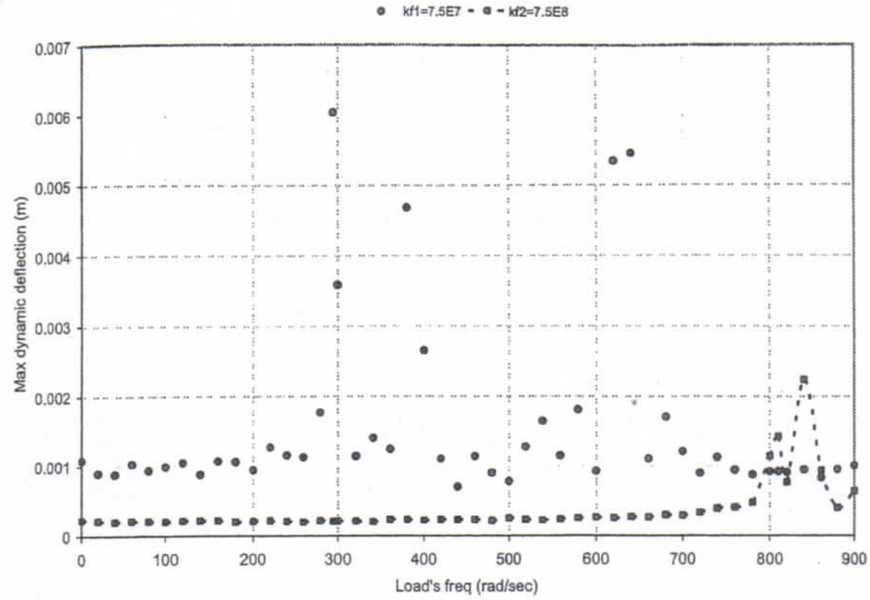

Figure 10. Response spectra as a function of load's frequency for various values of elastics foundation stiffness.

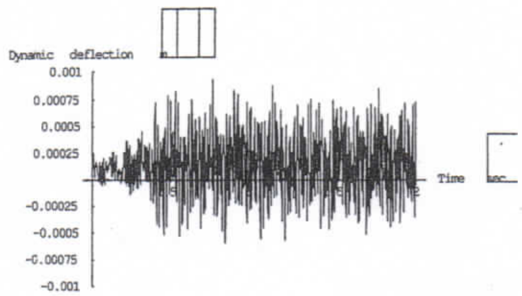

Figure 11. Time history at central point for k_f=7.5x10^7 N/m^3, γ=0%, ω=50 rad/s.

Figure 12. Time history at central point for k_f=7.5x10^8 N/m^3, γ=0%, ω=50 rad/s.

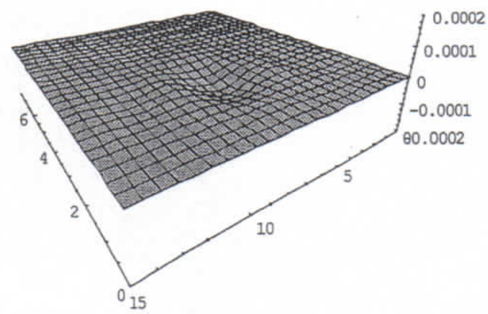

Figure 13. Total dynamic deflection shape calculated at $t_0 \le t \le t_1$, k_f=7.5x10^7 N/m^3, γ=10%, ω=50 rad/s.

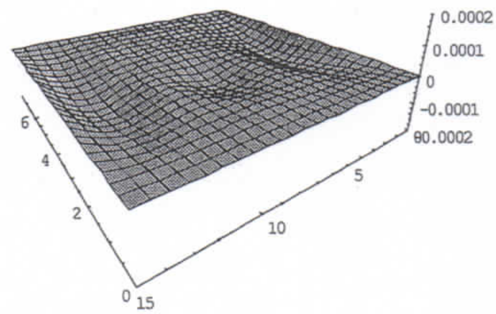

Figure 14. Total dynamic deflection shape calculated at $t_1 \le t \le t_2$, k_f=7.5x10^7 N/m^3, γ=10%, ω=50 rad/s.

6. CONCLUSION

In this research work, a procedure is proposed, using the Modified Bolotin Method and the Duhamel integration method to investigate the dynamic behavior of a damped orthotropic plate resting on an elastic Pasternak foundation subjected to a dynamic moving load. The effect of the elastic foundation stiffness, magnitude of the sudden x-position change and damping ratio are discussed. Numerical examples show that the elastic foundation stiffness and damping ratio are the main factors having significant effects on the dynamic responses. The maximum dynamic deflections increase as the magnitude of the sudden longitudinal position change increases for all loading frequencies studied. The absolute maximum dynamic deflections will occur if the time of change in longitudinal position of the load coincides with the time at which the deflection is maximal prior to the sudden change in longitudinal position.

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