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DYNAMICS RESPONSE OF STIFFENED ORTHOTROPIC PLATE SUBJECTED TO BLAST LOADING

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Abstract

An investigation has been conducted to examine the dynamic behavior of an orthotropic plate with different stiffener configurations, subjected to a localized blast load. The orthotropic plates are 4.5 m by 5.5 m, and 0.12 m thick, with partially fixed boundary conditions at all edges. The aims of this work is to determine the dynamic response of orthotropic plates with different stiffener configurations to a localized blast load and considering the effect of damping ratio, position of the localized blast load and loading duration. Natural frequencies of the system are solved numerically by using the Modified Bolotin Method, while the wave numbers are obtained from two transcendental equations. The localized blast loading is expressed by using Dirac delta function, which is further integrated by using the Duhamel integration method to find the dynamic response of the system. Special emphasis is focused on the dynamic deflection of the mid-point of the orthotropic plates, bending moments and vertical forces of the system. The results obtained give an insight into the effect of stiffener configurations and of the above parameters on the response of the orthotropic plates under the localized blast loading and indicate that stiffener configurations, time duration, location of the localized blast loading and damping ratio can affect their overall behavior.

Keywords: localized blast load, orthotropic plate, stiffeners, damping ratio, time duration, Modified Bolotin Method, transcendental equations.

1. Introduction

The response of structural components subjected to blast loading has been the subject of a great deal of research. To provide adequate protection against blast loading and explosion, the design of civil engineering structures and public buildings such as schools, and hospitals should receive more attention from structural engineers. Louca and Harding [1], Kadit et al [2] had presented analyses for plates subjected to blast loading, while Kadid [3] had presented a numerical study of stiffened plates to uniform blast loading, which included the effect of the stiffeners configurations to the plate's response. Dobyns [4] had presented analyses of simply supported orthotropic plates subject to static and dynamic loads, which included the numerical solution of plate to blast loads that was modeled as a triangular function, an exponential function and a stepped triangular function. Papers dealing with the response of orthotropic damped plates to blast loading with an irregular support condition were far more complicated to be numerically solved. What is the most relevant to the present work is the use of the Modified Bolotin Method for solving the vibration modes of rectangular plates [5]. Previous extensive studies on the dynamic response of damped orthotropic plates with a very general restraint condition along its support and unsymmetrical boundary conditions had been conducted Alisjahbana and Wangsadinata [6].

In the present research work the problem of an orthotropic stiffened plate under a localized bl load is further studied, whereby the plate is partially fixed along its supports. The vibration modes a solved using the Modified Bolotin Method. As the mode shapes are expressed as a product eigenfunctions, the dynamic solution of the plate is expressed in integral from, so that it can be read integrated to obtain various dynamic responses of the plate.

The geometry and material properties are assumed to be linear elastic and the orthotrop stiffened plate under consideration is of finite dimensions. Finally results for dynamic responses su as mid-point deflection, bending moments and vertical forces of the plate are presented incorporatir the effects of stiffeners configurations, the effect of damping ratio, location and time duration of localized blast load.

1. Governing equations

Using the classical theory of thin plates, the equation of equilibrium of an elastic orthotropic stiffene plate is as follows:

$$D_x \frac{\partial^4 w(x,y,t)}{\partial x^4} + 2B \frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w(x,y,t)}{\partial y^4} + \gamma h \frac{\partial w(x,y,t)}{\partial t} + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = p(x,y,t) \tag{1}$$

$$D_x = \frac{E'h^3}{12} + \frac{E_x b_x}{6a_x}\left(\left(h_x - \left(e_x - \frac{h}{2}\right)\right)^2 (2h_x + e_x + h) - \left(e_x - \frac{h}{2}\right)^2 (e_x + h)\right) \tag{2}$$

$$D_y = \frac{E'h^3}{12} \tag{3}$$

where D_x and D_y are the flexural rigidities in x and y direction respectively, B the torsional rigidity, γ the damping ratio, ρ mass density of the plate. The plate is stiffened by stiffeners of width b_x and height h_x. The origin of the Cartesian coordinates (x,y) is set at the upper left corner of the plate. w(x,y,t) is the transverse deflection of the mid surface. The two considered types of support conditions for each plate edge are as follow:

Along x=0 and x=a

$$-D_x\left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \nu_y \frac{\partial^2 w(x,y,t)}{\partial y^2}\right) = k_1 \frac{\partial w(x,y,t)}{\partial x}; \quad w(x,y,t) = 0 \tag{4}$$

Along y=0 and y=b

$$-D_y\left(\frac{\partial^2 w(x,y,t)}{\partial y^2} + \nu_x \frac{\partial^2 w(x,y,t)}{\partial x^2}\right) = k_2 \frac{\partial w(x,y,t)}{\partial y}; \quad w(x,y,t) = 0 \tag{5}$$

where k_1 is an elastic rotational restraint constant along x=0 and x=a and k_2 is an elastic rotational restraint along y=0 and y=b. A model of an orthotropic stiffened plate with rotational restraints along its edges subjected to a blast loading can then be established.

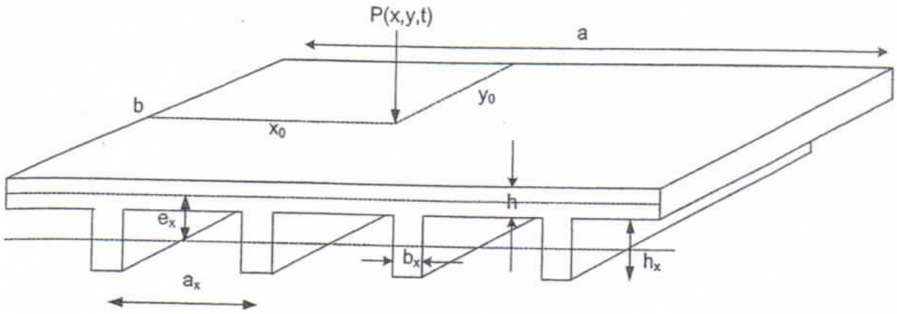


Figure 1. Rectangular orthotropic stiffened plate subjected to dynamic load p(x,y,t).

A localized blast load modeled as a triangular pulse function can be expressed by the following expression:

$$p(x,y,t) = P(t)\delta[x - x(t)]\delta[y - y(t)] = P_0\left(1 - \frac{t}{t_d}\right)\delta[x - x_0]\delta[y - y_0] \tag{6}$$

$$P(t) = P_0(1 - \frac{t}{t_d}) \text{ for } 0 \le t \le t_d \tag{7}$$

$$P(t) = 0 \text{ for } t > t_d \tag{8}$$

where P_0 =the maximum amplitude of the load; t_d= time duration; x_0= position of the localized blast load in x direction; y_0= position of the localized load in y direction.

2. General Analysis

In order to solve the problem described above, it is assumed that the principal elastic axes of the material are parallel to the plate edges and the free vibration solution of the problem is set as:

$$w(x,y,t) = W(x,y)\sin\omega t \tag{9}$$

where ω is the circular frequency and W(x,y) is a function of the position coordinates only. Then substituting Eq.(7) into the undamped free vibration form of Eq.(1) yields:

$$D_x\frac{\partial^4 W(x,y)}{\partial x^4} + 2B\frac{\partial^4 W(x,y)}{\partial x^2\partial y^2} + D_y\frac{\partial^4 W(x,y)}{\partial y^4} - \rho h\omega^2 W(x,y) = 0 \tag{10}$$

The next step is to find the solution of Eq.(10) with the boundary conditions according to Eq (4) and Eq.(5), to obtain the eigen frequencies and the mode shapes of the orthotropic plate with mixed support conditions at its edges. By postulating the following eigen frequency, which is analogous to the case of a plate simply supported at all edges [5], natural frequencies of the system can be expressed as:

$$\omega_{mn}^{\ 2} = \left(\frac{\pi^4}{\rho h}\right)\left[D_x\left(\frac{p}{a}\right)^4 + 2B\left(\frac{pq}{ab}\right)^2 + D_y\left(\frac{q}{b}\right)^4\right] \tag{11}$$

where p and q are real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method [5].

3. Dynamic Response of the Plate

The dynamic response of the stiffened orthotropic plate can be found by using the method of variation of parameters as a general method of determining a particular solution of the corresponding non-homogeneous partial differential equation, which can be written in the following form:

$$w_{mn}(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}X_m(x)Y_n(y)T_{mn}(t) \tag{12}$$

where $X_m(x)$, $Y_n(y)$ are eigenfunctions, $T_{mn}(t)$ is a function of time, which must be determined through further analysis.

The differential equation for the coefficient functions $T_{mn}(t)$ can be expressed as:

$$\ddot{T}_{mn}(t)+2\gamma\omega_{mn}\dot{T}_{mn}(t)+\omega_{mn}^2T_{mn}(t)=\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\frac{p(x,y,t)}{\rho hQ_{mn}} \tag{13}$$

where Q_{mn} is a normalization factor.

The particular solution of the temporal function $T_{mn}(t)$ can be represented in a form of the Duhamel convolution integral as follows:

$$T^{*}_{mn}(t)=\int_0^t\left[\frac{p(x,y,\tau}{\rho hQ_{mn}}\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\right]\left[\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}}\sin\left(\omega_{mn}\sqrt{(1-\gamma^2)}\,(t-\tau)\right)\right]d\tau \tag{14}$$

The general solution for the forced response deflection of the plate to an arbitrary dynamic moving load $p(x,y,t)$ is given in integral form as follows:

$$w(x,y,t)=\sum_{m=1}^{m=\infty}X_m(x)\sum_{n=1}^{n=\infty}Y_n(y)\int_0^t\left[\frac{p(x,y,\tau}{\rho hQ_{mn}}\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\right]\left[\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}}\sin\left(\omega_{mn}\sqrt{(1-\gamma^2)}\,(t-\tau)\right)\right]d\tau \tag{15}$$

Bending moments and vertical shear forces in the plate can be computed in terms of the deflection and its derivatives obtained from Eq. (15).

4. Results

A reinforced concrete rectangular damped plate stiffened by rectangular stiffeners along the x axes is considered. The material is assumed to be orthotropic and linearly elastic. The data for the plate and blast load are: a=4.5 m, b=4.5 m, h=0.12 m, E_c=2.5743x10^{10} N/m^2, υ = 0.2, b_x=0.2 m, e_x=0.12 m, h_x= 0.24 m, ρ= 2400 kg/m^2, and k_1= 5x10^6 Nm/rad/m, k_2= 5x10^8 Nm/rad/m. The boundary conditions are transversely supported edges with constant elastic rotational restraint along the x and y edges. In the following discussion x_0=a/3 and y_0=b/3 refering to the positions of the blast loading, t_d refers to the time duration of the blast loading.

The absolute maximum dynamic deflection at the mid plate due to a blast load has been calculated by using 5 modes in the x and y directions (m=1,2,...,5 and n=1,2,...,5). The blast load is modeled as a triangular function, where P_0= 1.3x10^6 N/m^2. In order to study the effect of time duration, 3 time intervals t_d have been used in this study, namely 1ms, 2 ms and 20 ms.

Two transcendental equations are used to obtain the values of p and q and the natural frequencies of the plate for two models; model 1 (1 stiffener) and model 2 (2 stiffeners). The natural frequencies of the systems for the first 3 modes are shown in Table 1.

4.1. Effect of time duration

The absolute maximum dynamic deflection has been computed for 3 different damping factors and 3 different time durations of the blast loading, the results of which are shown in Table 2.

For model 1 (1 stiffener) increasing the time duration by factors of 2 and 20 for the undamped system (γ=0%) has resulted in an increase in the mid-plate displacement by a factor of 1.87 and 9.74. For model 2 (2 stiffeners) increasing the time duration by factors of 2 and 20 for the undamped system (γ=0%) has resulted in an increase in the mid-plate displacement by a factor of 1.97 and 9.35. Therefore, the time duration of the blast loading plays an important role in determining the level of response of the plate.

4.2. Effect of Stiffeners Configurations

For the time duration t_d=20 ms (Table 2), the introduction of stiffeners decreases the mid-plate displacement; in model 1 it is 0.00368568 m, while in model 2 it is slightly smaller, namely

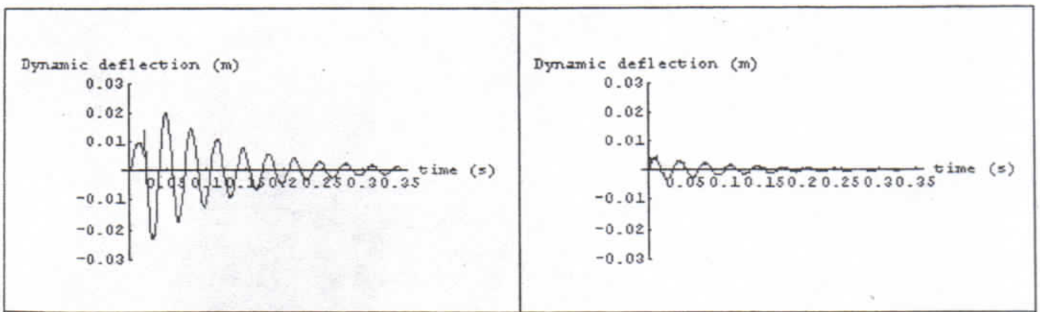
0.00313621 m. Thus the configuration of stiffeners can have an important influence on the response of the orthotropic stiffened plates. The same conclusions apply to t_d=1 ms, and 2 ms.

Table 1. Fundamental frequencies for the first 3 modes in x direction (m=1, 2, 3) and y direction (n=1, 2, 3).

m	n	Model 1 (1 stiffener)			Model 2 (2 stiffeners)		
		p	q	ω_{mn} (rad/s)	p	q	ω_{mn} (rad/s)
1	1	0.97947	1.34024	177.068	0.987437	1.32382	189.8848
	2	0.992912	2.42782	384.752	0.995349	2.41899	396.2983
	3	0.996948	3.4605	691.32	0.997938	3.45523	702.75478
2	1	1.991	1.18823	434.955	1.99486	1.17558	492.17595
	2	1.99418	2.30921	628.997	1.99649	2.29489	682.51781
	3	1.99652	3.37736	932.487	1.9978	3.36543	983.6299
3	1	2.99558	1.12412	890.911	2.99751	1.11558	1021.1863
	2	2.99639	2.2261	1070.93	2.99791	2.21307	1197.4951
	3	2.9973	3.30043	1363.82	2.99838	3.28664	1486.42023

Table 2. The absolute maximum dynamic deflection of a damped orthotropic stiffened plate subjected to a localized blast load for different values of damping ratio and stiffeners configuration.

Damping ratio (γ)	t_d=1 ms w_{max} (m)	t_d= 2 ms w_{max} (m)	t_d= 20 ms w_{max} (m)
Model 1 (1 stiffener)			
γ=0 %	0.00378321	0.00706284	0.0368568
γ=5 %	0.00278386	0.00533084	0.026842
γ= 10 %	0.0022917	0.00450229	0.0212333
Model 2 (2 stiffeners)			
γ=0 %	0.00335309	0.0066007	0.0313621
γ=5 %	0.00242794	0.00472416	0.0229505
γ= 10 %	0.00197927	0.00394233	0.0133791



Model 1 (1 stiffener) | Model 2 (2 stiffeners)

Figure 2. Dynamic deflection time history stiffened orthotropic plates with 1 stiffener (model 1) and 2 stiffeners (model 2) subjected to localized blast load.

4.3. Dynamic deflection time history

In Figure 2 the mid-plate deflection time histories are shown for γ=5% and t_d= 20 ms each for model 1 and model 2. It can be seen, that the dynamic deflection of model 2 is relatively smaller than that of model 1 due to the greater stiffeners effect. It can also be seen, that deflections are decaying with time due to the damping effect.

4.4. Dynamic bending moments

From the deflection w(x,y,t) and its derivatives, bending moments and shear forces have bee computed. In Figure 3 the M_x and M_y distribution and shape are shown for model 2 with γ=5% an t_d=20 ms at a certain point of time. At other points in time the distribution and shape will be different.

4.5. Distribution of bending moments and shear forces along the coordinate axes

Figure 5 shows the distribution along the x and y axes of the bending moments M_x and M_y and th shear forces q_x and q_y, which in fact are support reactions of the plate along the edges. Suppor reactions are as important as mid-plate forces, as failure may occur at mid-plate or at the supports.

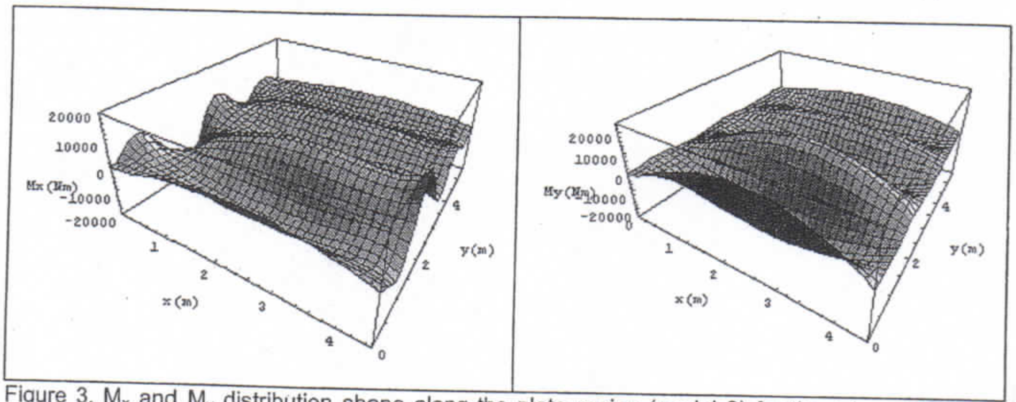


Figure 3. M_x and M_y distribution shape along the plate region (model 2) for the value of t_d=20 ms, γ=5%.

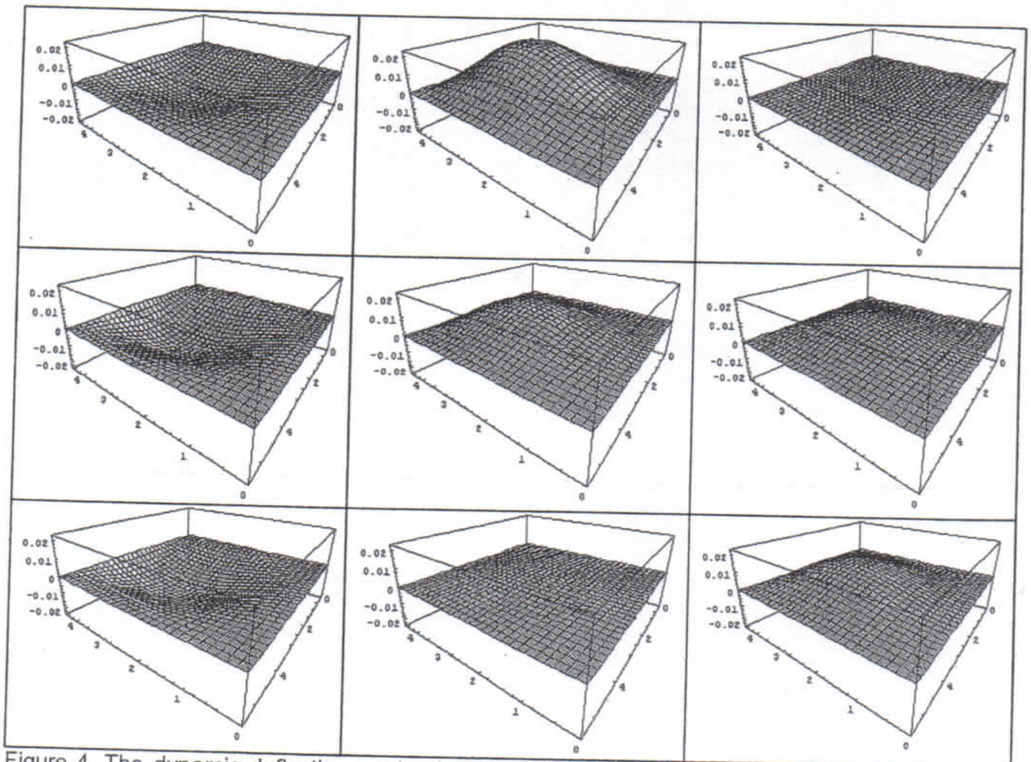
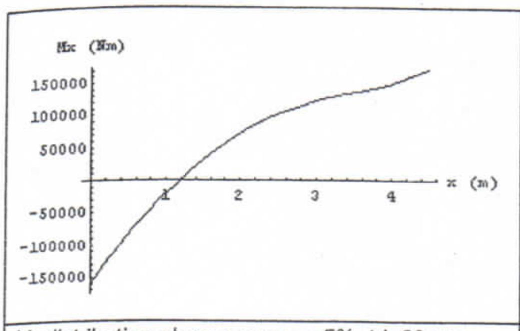
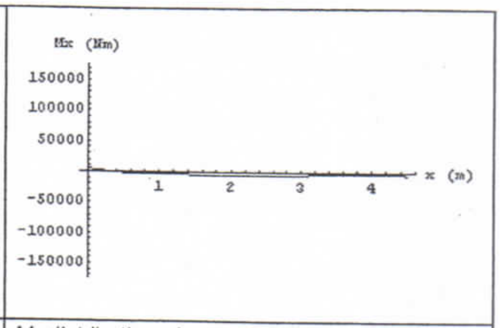


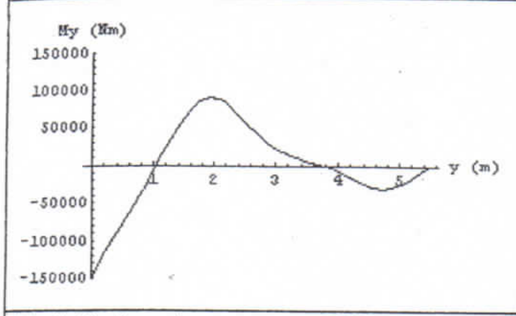
Figure 4. The dynamic deflection mode shapes of the undamped stiffened orthotropic plate with 2 stiffeners (model 2) subjected to a blast loading, γ=5%, t_d=20 ms.



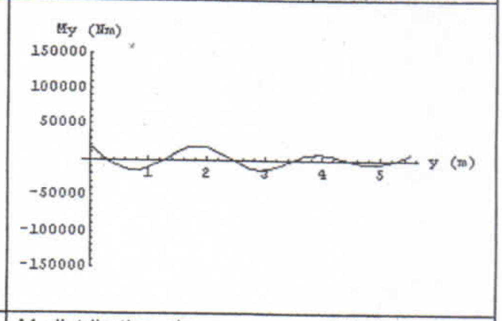
M_x distribution along x axes, γ=5%, td=20 ms.



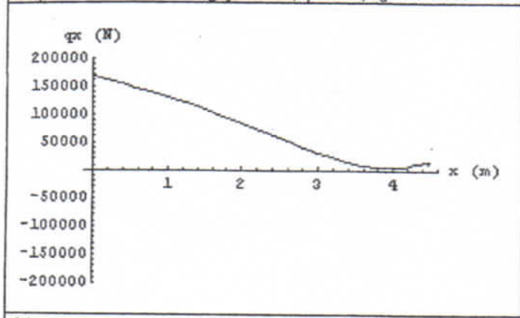
M_x distribution along x axes, γ=5%, td=2 ms.



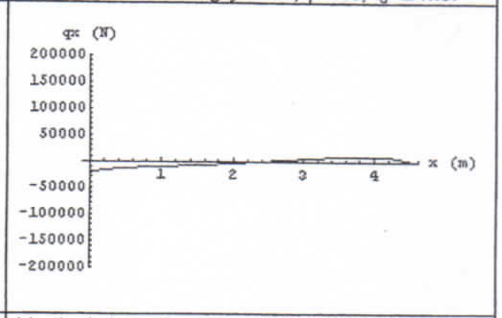
M_y distribution along y axes, γ=5%, t_d=20 ms.



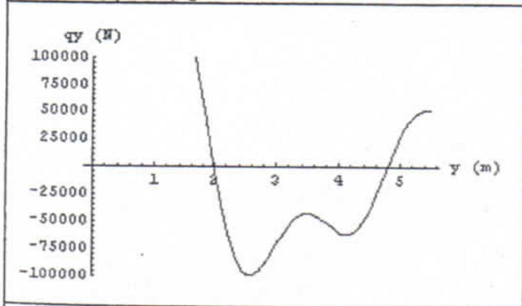
M_y distribution along y axes, γ=5%, t_d=2 ms.



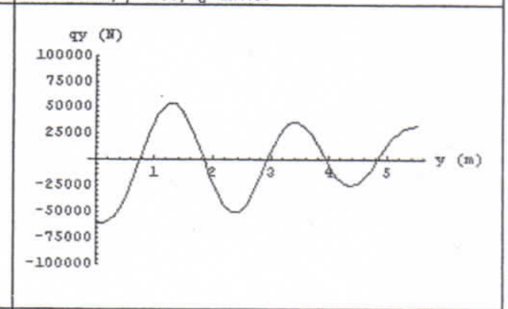
Vertical shear force (q_x) distribution along x direction, γ=5%, t_d=20ms.



Vertical shear force (q_x) distribution along x direction, γ=5%, t_d=2ms.



Vertical shear force (q_y) distribution along y direction, γ=5%, t_d=20ms.



Vertical shear force (q_y) distribution along y direction, γ=5%, t_d=2ms.

Figure 5. Various response of the damped orthotropic plate with 1 stiffener (model 1) subjected to a blast loading.

5. Conclusions

From the dynamic analyses carried out to examine the behavior of partially-fixed stiffened orthotropic plates under a localized blast loading, the following conclusions can be drawn:

1. The effect of stiffeners configurations can be very important, since it affects drastically the overall behavior of the stiffened orthotropic plate.
2. The time duration is one of the most important parameter, since it has an influence on other responses, such as moment distribution, shear distribution and the maximum dynamic deflection of the system.
3. The inclusion of damping in calculating the dynamic response of the system results in a much stiffer response, especially for large values of t_d, resulting in lower mid-plate dynamic deflection.

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