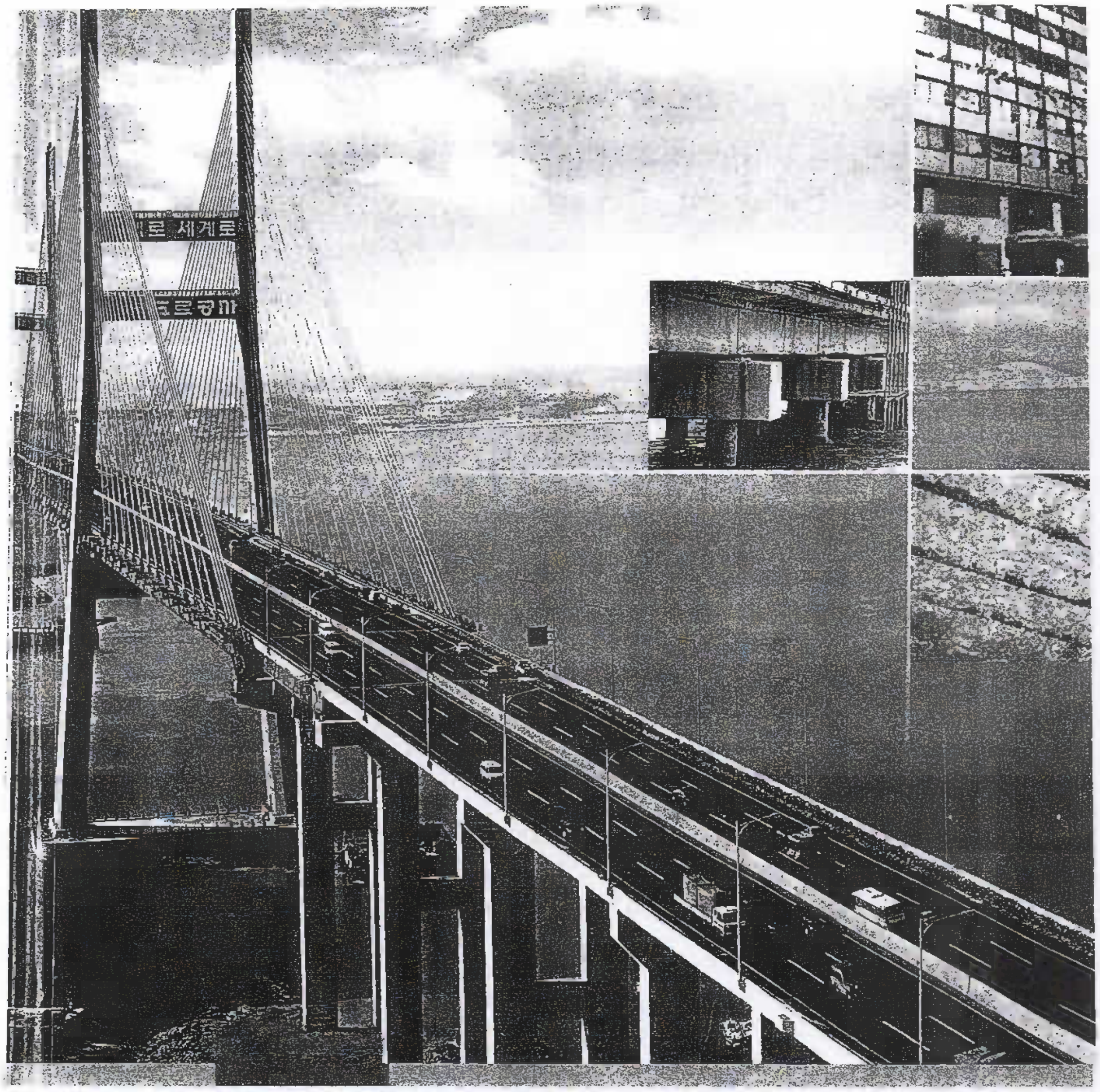

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DYNAMICS OF RIGID ROADWAY PAVEMENTS

S.W. Alisjahbana, W. Wangsadinata
Tarumanagara University, Indonesia

Abstract

In this paper the dynamic response of rigid roadway pavements to moving dynamic loads is investigated. To solve this problem, the rigid pavement is modelled as a rectangular damped orthotropic plate resting on a continuous elastic foundation. Assuming the plate to be simply supported, the natural frequencies are computed, whereby the wave numbers are mπ/a and nπ/b, 'a' and 'b' denoting the length of the plate in the x and y direction and m and n being positive integers, determining the mode number. The mode shape is presented as a product of eigenfunctions. The dynamic loading function is described as a concentrated load of harmonically varying magnitude, travelling with a constant speed. Such a loading may be considered representing a truck wheel load moving on a roadway pavement. The general solution for this loading function is derived in integral form, which is then solved to obtain the forced responses of the plate. The purpose of this paper is to illustrate and demonstrate the applicability of this theory by presenting the analysis of the natural frequencies of an example rigid roadway pavement and its dynamic response deflections, bending moments and shear forces due to the dynamic loading of a passing truck.

1. Introduction

Numerous plate elements used in civil engineering, aerospace and marine structures are supported by elastic or viscoelastic media and subjected to dynamic loads. The usual approach in formulating these problems is based on the inclusion of the foundation reaction into the corresponding differential equation of the plate. The foundation is very often a complex medium, but since of interest here is the response of the plate, the problem reduces to finding a relatively simple mathematical expression, describing the response of the foundation at the contact area. The simplest representation of a continuous elastic foundation had been provided by Winkler [1] by assuming it to consist of closely spaced independent linear springs. It presumes a linear force-deflection relationship, so that if a deflection w is imposed on the foundation, it resists with a pressure k_1w, where k_1 is the foundation modulus. Some of the more recent studies dealing with the stability and the dynamic response of an orthotropic plate included work by Paliwal & Gohsh [2], who determined the stability of orthotropic plates on a Kerr

; of a rectangular orthotropic .na [4] presented the analysis cluded the effect of in-plane

plate are parallel to the x and bjected to a general moving ith a foundation modulus k_1. can be expressed as follows:

$$\frac{\partial^2 w(x,y,t)}{\partial t^2} + k_1 w = p(x,y,t) \quad (1)$$

respectively, B the effective lensity. The solution of the the method of separation of fy the boundary conditions

$$\frac{\pi x}{a}\Big)\sin\left(\frac{n\pi y}{b}\right)T_{mn}(t) \quad (2)$$

e obtains:

$$x, y)T_{mn}(t) =$$

$$) = \beta^4_{mn} \quad (3)$$

(t) depends on the temporal hese separation constants, or ; follows:

$$(4)$$

e, which are related to the

$$(5)$$

essed as

$$+ b_{omn} e^{-i\sqrt{1-\bar{\gamma}^2}\,\omega_{mn} t}\Big] \quad (6)$$

onditions.

3. Forced response

Since a fundamental set of solutions of the homogeneous partial differential equation is known and given by the eigenfunctions, it is appropriate to use the method of variation of parameters as a general method of determining a particular solution of the corresponding non-homogeneous partial differential equation. Using the characteristic function from Eq.(2), an appropriate solution for the forced response may be written in the form:

$$w_{mn}(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\left[\frac{m\pi x}{a}\right]\sin\left[\frac{n\pi y}{b}\right]T_{mn}(t) \quad (7)$$

where T_{mn} (t) is a function of time, which must be determined through further analysis. After substituting Eq.(7) into Eq.(1) the governing non-homogeneous partial differential equation can be put in the following form:

$$\left[D_x \frac{m^4\pi^4}{a^4} + 2B\frac{m^2 n^2 \pi^4}{a^2 b^2} + D_y \frac{n^4\pi^4}{b^4} + k_1\right]W_{mn}(x,y)T_{mn}(t) +$$

$$+\rho h \frac{\partial^2 T_{mn}(t)}{\partial t^2} W_{mn}(x,y) + \gamma h \frac{\partial T_{mn}(t)}{\partial t} W_{mn}(x,y) = p(x,y,t) \quad (8)$$

The differential equation for the coefficient functions $T_{mn}(t)$ may be obtained by multiplying both sides of Eq.(8) in turn by either sin [mπx/a] or sin [nπx/b] and integrating over the plate region 0<x<a;0<y<b. Thus an ordinary differential equation for $T_{mn}(t)$ is obtained in the following form:

$$\ddot{T}_{mn}(t) + 2\bar{\gamma}\omega_{mn}\dot{T}_{mn}(t) + \omega^2_{mn} T_{mn}(t) = \left[\int_0^a \sin\frac{m\pi x}{a}dx \int_0^b \sin\frac{n\pi y}{b}dy\right]\frac{p(x,y,t)}{\rho h Q_{mn}} \quad (9)$$

where $\bar{\gamma} = [\gamma/2\rho\omega_{mn}]$ is a damping factor ratio and Q_{mn} a normalization factor. Note that the homogeneous solution of Eq.(9) is identical with the one previously obtained using the separation of variables solution method. The total solution of Eq.(9) is then

$$T_{mn}(t) = \hat{T}_{mn}(t) + T^*_{mn}(t) \quad (10)$$

where $\hat{T}_{mn}(t)$ is the homogeneous solution and $T^*_{mn}(t)$ the particular solution that can be represented in a form of the Duhamel convolution integral as follows

$$T^*_{mn}(t) = \int_0^t \left[\frac{p(x,y,\tau)}{\rho h Q_{mn}} \int_0^a x_m(x)dx \int_0^b y_n(y)dy\right]\left[\frac{e^{-\bar{\gamma}\omega_{mn}(t-\tau)}}{\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}} \sin\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}(t-\tau)\right]d\tau \quad (11)$$

The homogeneous solution $\hat{T}_{mn}(t)$ contains constants that must be determined from the initial conditions, representing a transient state of vibration. The nature of the steady state forced responses of the plate is contained entirely in the functions $T^*_{mn}(t)$ defined by Eq.(11). Substituting the expressions for the coefficient functions in Eq.(11), the general solution for the forced response deflection of the plate to an arbitrary dynamic load p(x,y,t) is given in integral form by

$$ {}_{mn} e^{-i\sqrt{1-\bar{\gamma}^2}\,\omega_{mn} t} \Big] \Big] + $$

$$ \text{in} \sqrt{1-\bar{\gamma}^2}\,\omega_{mn}(t-\tau) \Big] \Big] d\tau \quad (12) $$

determine the response of the
load of harmonically varying
;ht line path at a constant x
ɜl load moving on a roadway

(13)

y of the harmonic variation.
.q.(12) the general deflection

$$ + b_{mn} e^{-i\sqrt{1-\bar{\gamma}^2}\,\omega_{mn} t} \Big] \Big] + $$

$$ y - v\tau \big] dxdy \Big] \Big] x $$

(14)

$$ \text{os}(\omega t)]\sin\frac{m\pi x_o}{a}\sin\left[\frac{n\pi}{b}v\tau\right] \quad (15) $$

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s leaving the plate. Thus, this
·lves a harmonically varying
position. The second part, in
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ıt t = t_o determines the initial

conditions for the second part of the problem. The response deflection of the system can be computed from the following equation:

$$ w_{mn}(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-\bar{\gamma}\omega_{mn}(t-t_o)} \left[w_{omn}\cos\left[\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}(t-t_o)\right] + \frac{v_{omn}+\bar{\gamma}\omega_{mn}w_{omn}}{\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}} \sin\left[\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}(t-t_o)\right] \right] \quad (16) $$

in which w_{omn} and v_{omn} are the initial deflection and velocity at $t = t_o$. Bending moments and vertical shear forces in the plate can be computed in terms of the deflection and its derivatives obtained from Eq.(16) from the following expressions:

$$ M_X = -\left[D_X \frac{\partial^2 w}{\partial x^2} + B\frac{\partial^2 w}{\partial y^2}\right] \quad ; \quad M_Y = -\left[D_Y \frac{\partial^2 w}{\partial y^2} + B\frac{\partial^2 w}{\partial x^2}\right] $$

$$ Q_X = -\frac{\partial}{\partial x}\left[D_X \frac{\partial^2 w}{\partial x^2} + H\frac{\partial^2 w}{\partial y^2}\right] \quad ; \quad Q_Y = -\frac{\partial}{\partial y}\left[D_X \frac{\partial^2 w}{\partial y^2} + H\frac{\partial^2 w}{\partial x^2}\right] \quad (17) $$

where H=B+2G and G is the elastic shear modulus of the plate. The flexural rigidities and the effective torsional rigidity can be expressed as follows :

$$ D_x = \frac{E_x h^3}{12(1-v_x v_y)} \quad ; \quad D_y = \frac{E_y h^3}{12(1-v_x v_y)} \quad ; \quad B = \sqrt{D_x D_y} \quad (18) $$

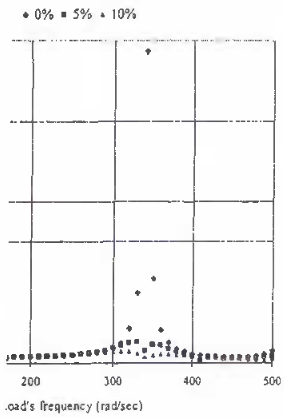
where E_x and E_y are the elasticity moduli, while v_x and v_y the Poison's ratios in the x and y direction respectively and h the thickness of the plate.

5. Numerical example

Using the procedure described above, a roadway pavement subjected to the moving dynamic wheel load of a truck will be analysed. The effect of changing the load's angular frequency ω and the damping ratio γ will be investigated. The average wheel load is $P_o = 10^4$ N, travelling with a constant speed v = 60 km/hr along the y direction. The following numerical results have been calculated for the following case: a = 7m, b = 20m, $\rho = 2.4\text{x}10^3$ kg/m^3, h = 0.35m, $E_x = 30\text{x}10^9$N/m^2, $E_y = 20\text{x}10^9$N/m^2, $v_x = 0.2$, $v_y = 0.1$, G = 10^{10}N/m^2, $k_1 = 7.5\text{x}10^7$ N/m^2/m, $x_o = 3.5$ m.

Table 1. Natural frequencies of the plate for the first 5 modes

n	m=1	n	m=2	n	m=3	n	m=4	n	m=5
	ω_{mn} (rad/sec)		ω_{mn} (rad/sec)		ω_{mn} (rad/sec)		ω_{mn} (rad/sec)		ω_{mn} (rad/sec)
1	340.582	1	442.833	1	734.738	1	1209.36	1	1845.46
2	424.688	2	514.072	2	783.805	2	1243.36	2	1870.97
3	536.314	3	614.723	3	859.38	3	1298.05	3	1912.73
4	661.676	4	732.765	4	955.192	4	1370.95	4	1969.71
5	794.296	5	861.076	5	1065.8	5	1459.34	5	2040.63



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loading. By comparing the
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Figure 4 gives an overview
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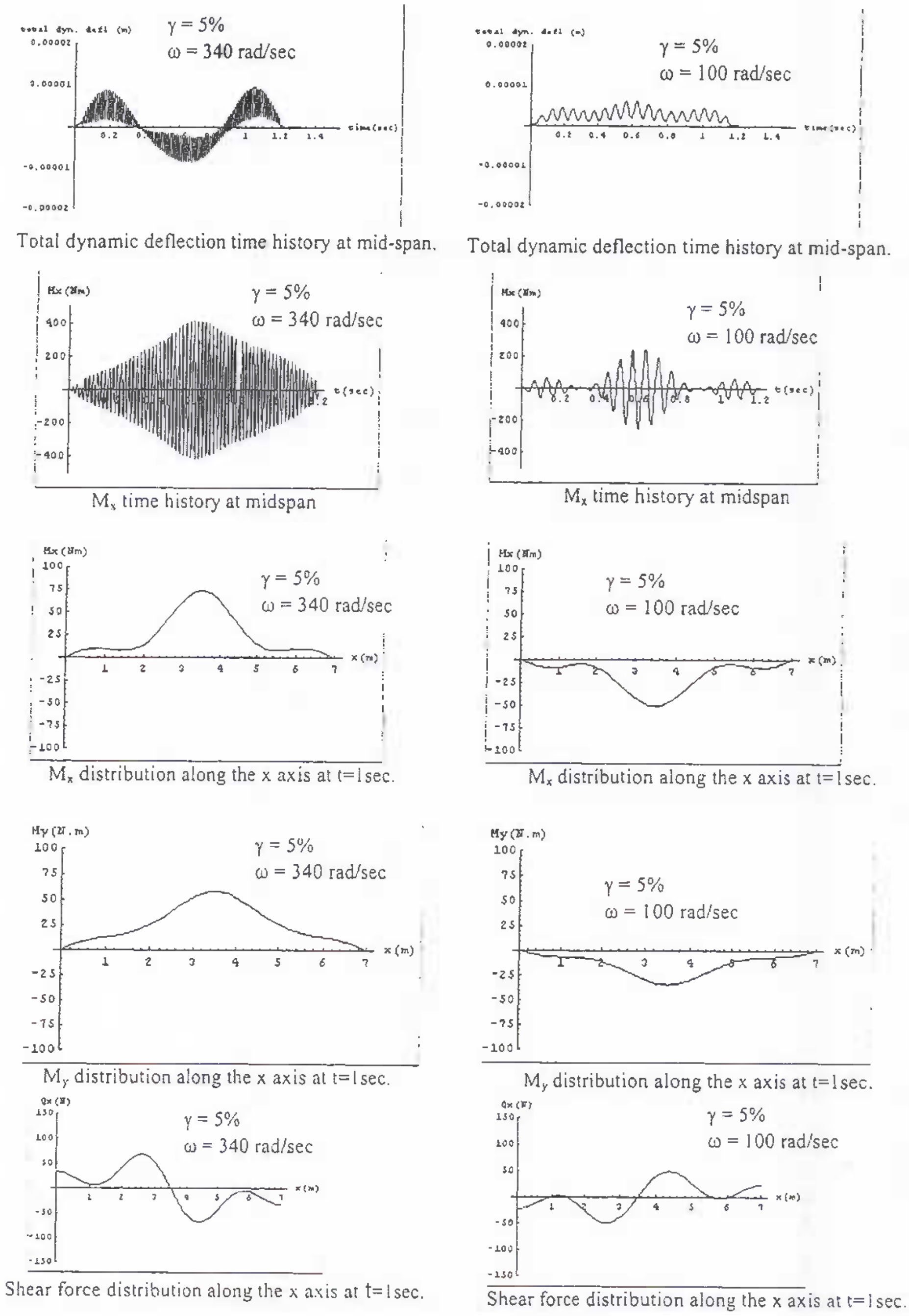

Figure 3. Dynamic responses of the plate at near resonance condition (left) and away from it (right).

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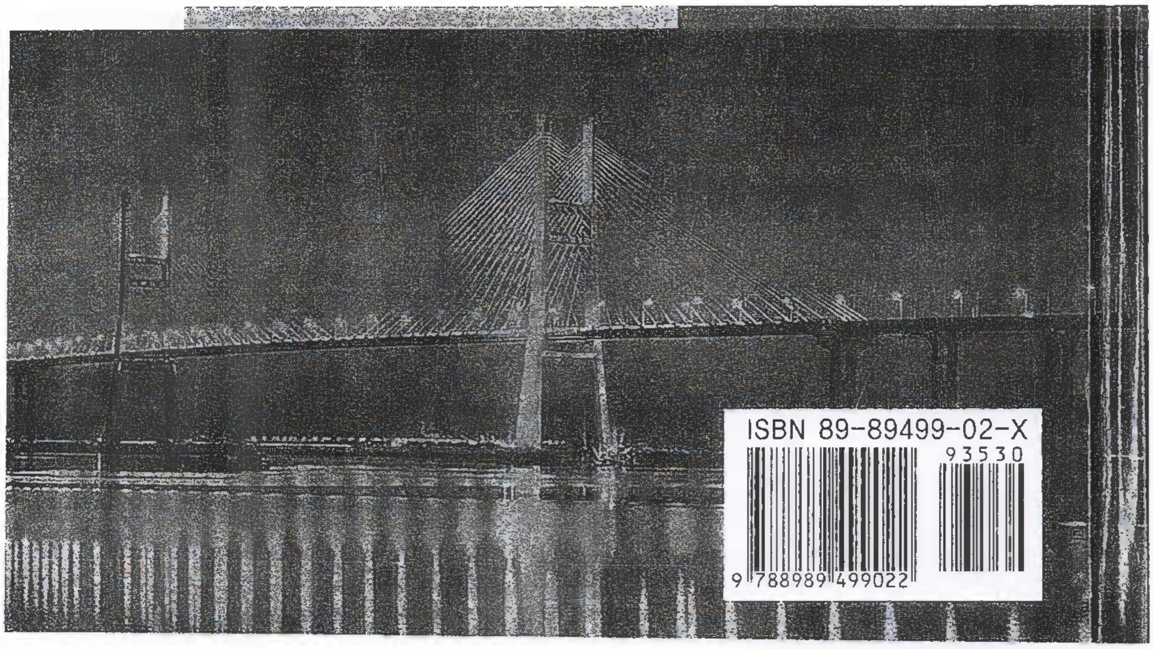

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