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DYNAMIC RESPONSE OF DAMPED ORTHOTROPIC PLATES ON A KERR FOUNDATION

Sofia W. ALISJAHBANA

Tarumanagara University, Jakarta 11440,
INDONESIA
E-mail:wangsadi@indosat.net.id

ABSTRACT

Several of the plate structures used in civil engineering, aerospace and marine construction are supported by elastics or viscoelastic media and subjected to both in-plane forces and lateral forces. The usual approach in formulating these problems is based on the inclusion of the foundation reaction into the corresponding differential equation of the plate. It is the purpose of this analysis to present a general solution based on double Fourier techniques for the free and forced response of damped orthotropic plates on Kerr foundation subjected to an arbitrary surface load p(x,y,t), including the effects of in-plane forces. General natural frequency equations are presented with numerical values included for the first five modes of vibration. Damping is included as a parameter in the formulation of this vibration problem to provide further understanding of its role in the dynamic response.

Nomenclature

a,b	=	Plate dimension in x and y direction
A_0, B_0	=	Constant coefficient
A_{mn}	=	Undetermined coefficients
c	=	Aspect ratio=a/b
D_x, D_y	=	Flexural rigidities in the x and y direction, respectively
B	=	$D_x \nu_y + \frac{Gh^3}{6}$
E_x, E_y	=	Young's moduli in the x and y direction, respectively
f_i	=	Frequency functions
G_s	=	Shear modulus for the shear layer
G_p	=	Modulus rigidity of orthotropic plate
h	=	Plate thickness
k_1, k_2	=	Foundation modulus of the upper and lower spring layer, respectively
m,n	=	Integer
$\bar{N}$	=	Nondimensional critical load identical to $Nb^2/\left[\pi^2\sqrt{D_x D_y}\right]$
N_x	=	In-plane load in the x direction
N_y	=	In-plane load in the y direction
p(x,y,t)	=	Dynamic transverse load
Q_{mn}	=	Normalization constant
w(x,y,t)	=	Plate deflection

α	=	Coefficient
β_{mn}	=	Separation of variable constant
γ	=	Damping coefficient
$\delta[.]$	=	Diract Delta function
ρ	=	Mass density
λ_1	=	Nondimanesional foundation modulus of the upper spring layer, identical to $k_1 b / \left[\pi^4 \sqrt{D_x D_y} \right]$
λ_2	=	Nondimanesional foundation modulus of the upper spring layer, identical to $k_2 b / \left[\pi^4 \sqrt{D_x D_y} \right]$
∇^2	=	Laplacian operator

1. Introduction

Rectangular orthotropic plates are extensively used in marine, aerospace, and civil engineering construction. Such plate structures are quite often subjected to in-plane forces and dynamic moving load. If the in-plane forces surpass a certain load intensity loss of stability and failure of the structure result. On the other hand if the natural frequencies of the system equal to the frequencies of the moving load the resonance condition result. Hence, it is essential to evaluate the critical values of the in-plane forces and the resonance conditions in the orthotropic plates.

Lekhnitskii [1] has described exhaustively the stability analysis of the orthotropic plates, employing static equilibrium and energy methods. Pathak has investigated the vibrations and buckling of orthotropic plates on a Pasternak foundation. The behaviour of many foundation materials with large void ratio and stiff clays cannot be represented by Winkler model. Paliwal [2] has described exhaustively the stability of the orthotropic plates on a Kerr foundation, which is one of the most advantages foundation model. Kneifati [3] has shown that the base response is more accurately represented by the Kerr model than the Winkler and Pasternak models. Herein, the author has made an attempt to study the buckling and the dynamic behaviours of a rectangular orthotropic plates subjected to in-plane and transverse moving load resting on a Kerr foundation model.

2. Analytical Formulation

Forced small amplitude vibrations of a thin elastic orthotropic damped plate on a Kerr foundation is governed by the linear partial differential equation:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} + \gamma h \frac{\partial w}{\partial t} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} = -p_1 \quad (1)$$

where p_1 is foundation response.

Because the Kerr foundation model consists of two spring layers and a shear layer with constants k_1, k_2 and G, the deflection of the plate is given by [4]

$$w(x, y, t) = w_1(x, y, t) + w_2(x, y, t) \quad (2)$$

The contact pressures under the plate (p_1) and the shear layer (p_2) are expressed by

$$p_1(x, y) = k_1 w_1 = k_1(w - w_2) \quad (3)$$

$$p_2(x,y) = k_2 w \tag{4}$$

On the other hand the governing differential equation for the shear layer is

$$k_2 w_2 - G\nabla^2 w_2 = p_1 \tag{5}$$

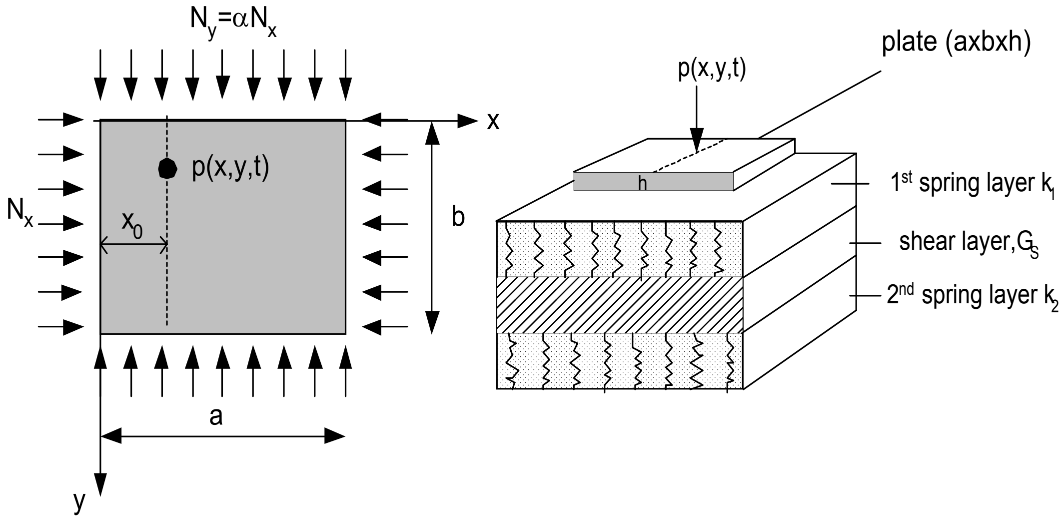

Figure 1. Rectangular orthotropic plate subjected to the bi-axial in-plane loads and dynamic transverse loads supported by a Kerr foundation.

Eliminating w_2 from Equations 3 and 5 and substituting the value of p_1 from Equation 1 into Equation 6, the governing differential equation of the rectangular damped orthotropic plate is

$$\begin{aligned}
&-\left[1+\frac{k_2}{k_1}\right]\left[D_x\frac{\partial^4 w}{\partial x^4} + 2H\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 w}{\partial y^4} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2}\right.\\
&\left.+\rho h\frac{\partial^2 w}{\partial t^2} + \gamma h\frac{\partial w}{\partial t} - p(x,y,t)\right] + \frac{G}{k_1}\left[D_x\left(\frac{\partial^6 w}{\partial x^6} + \frac{\partial^6 w}{\partial x^4 \partial y^2}\right) + 2H\left(\frac{\partial^6 w}{\partial x^4 \partial y^2} + \frac{\partial^6 w}{\partial x^2 \partial y^4}\right)\right.\\
&\left.+D_y\left(\frac{\partial^6 w}{\partial y^6} + \frac{\partial^6 w}{\partial x^2 \partial y^4}\right) + N_x\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2 \partial y^2}\right) + N_y\left(\frac{\partial^4 w}{\partial y^4} + \frac{\partial^4 w}{\partial x^2 \partial y^2}\right)\right] + \frac{G}{k_1}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\\
&\left(\rho h\frac{\partial^2 w}{\partial t^2} + \gamma h\frac{\partial w}{\partial t} - p(x,y,t)\right) = k_2 w - G\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)
\end{aligned} \tag{6}$$

By assuming the following deflection that satisfies the boundary conditions

$$W(x,y) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{7}$$

where A_{mn} are constant coefficients and m and n are integers and substituting the value of w from Equation 7 into Equation 6 and putting a/b=c, one obtains

$$\bar{N}\left[\left[\left(1+\frac{\lambda_2}{\lambda_1}\right)\left(\frac{m^2}{c^2}+\alpha n^2\right)\right]+\frac{\mu}{\lambda_1}\left[\frac{m^2}{c^2}+(\alpha+1)\frac{m^2n^2}{c^2}+\alpha n^4\right]\right]$$
$$=\lambda_2+\mu\left[\frac{m^2}{c^2}+n^2\right]+\left(1+\frac{\lambda_2}{\lambda_1}\right)\left[\sqrt{\frac{D_x}{D_y}}\frac{m^4}{c^4}+\frac{2H}{\sqrt{D_xD_y}}\frac{m^2n^2}{c^2}+\sqrt{\frac{D_y}{D_x}}n^4\right] \tag{8}$$
$$+\frac{\mu}{\lambda_1}\left[\sqrt{\frac{D_x}{D_y}}\left(\frac{m^6}{c^6}+\frac{m^4n^2}{c^4}\right)+\frac{2H}{\sqrt{D_xD_y}}\left(\frac{m^4n^2}{c^4}+\frac{m^2n^4}{c^2}\right)+\sqrt{\frac{D_y}{D_x}}\left(\frac{m^2n^4}{c^2}+n^6\right)\right]$$

Equation 8 gives all values of $\bar{N}$ corresponding to m=1,2,…, and n=1,2,…, as possible modes of plate buckling. For the case where forces N_x and N_y vary but maintain a certain constant ratio $N_x=N$ and $N_y=\alpha N$, the instability condition occurs when the plate exhibits one-half wave in the x direction and two-half waves in the y direction, that is m=1, n=2.

3. Natural Frequencies of the System

Free small amplitude vibrations of a thin, elastic orthotropic plate supported by a Kerr foundation are governed by the linear partial differential Equation 6 by letting the transverse load p(x,y,t) is equal to zero. When free vibrations are assumed, the motion is expressed as [5]

$$w(x,y,t)=W(x,y)T(t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}X_m(x)Y_n(y)T_{mn}(t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}T_{mn}(t) \tag{9}$$

where W(x,y) is the spatial function and $T_{mn}(t)$ is the temporal function.

For a plate with all sides simply supported the boundary conditions are:

$$W(x,y)=\nabla^2W(x,y)=0 \tag{10}$$

where ∇^2 is the Laplacian operator.

Substituting Equation 9 into the homogeneous form of Equation 6 gives the frequency of the system

$$(\omega_{mn})^2=-\left[\frac{\beta_{mn}k_1}{\rho h}\right]\left[G\left[\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right]+k_1+k_2\right]^{-1} \tag{11}$$

where β_{mn} are the separation constants that can be expressed by

$$\beta_{mn}=-\frac{G}{k_1}\left[D_x\left(\frac{m\pi}{a}\right)^6+(2H+D_x)\left(\frac{m\pi}{a}\right)^4\left(\frac{n\pi}{b}\right)^2+(2H+D_y)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^4+D_y\left(\frac{n\pi}{b}\right)^6\right]$$
$$-\frac{k_1+k_2}{k_1}\left[D_x\left(\frac{m\pi}{a}\right)^4+2H\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2+D_y\left(\frac{n\pi}{b}\right)^4-N\left[\left(\frac{m\pi}{a}\right)^2+\alpha\left(\frac{n\pi}{b}\right)^2\right]\right] \tag{12}$$
$$+N\frac{G}{k_1}\left[\left(\frac{m\pi}{a}\right)^4+(1+\alpha)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2+\alpha\left(\frac{n\pi}{b}\right)^4\right]-G\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]-k_2$$

4. Forced Response

Since a fundamental set of solutions of the free vibration problem is known and given by the eigen functions, it is appropriate to use the Lagrange's method [6] as a general method of determining a forced response to the corresponding non homogeneous partial differential equation of motion. It should be noted that while this method provides formula for computing the particular solution in a straightforward manner, it may not always be trivial to evaluate the resulting integrals in closed form.

Using the Fourier series representation for the plate transverse deflection given in Equation 9, the governing non homogenous partial differential equation of motion can be put in the form

$$\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} W_{mn}(x,y)\ddot{T}_{mn}(t) + \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} 2\xi\omega_{mn} W_{mn}(x,y)\dot{T}_{mn}(t)$$

$$+\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \frac{\beta_{mn} k_1 W_{mn}(x,y)T_{mn}(t)}{\rho h\left[G\nabla^2 W_{mn}(x,y) - \left(k_1+k_2\right)W_{mn}(x,y)\right]} = \frac{\left[\dfrac{G}{k_1}\nabla^2 - \dfrac{k_1+k_2}{k_1}\right]p(x,y,t)}{\rho h\left[\dfrac{G}{k_1}\nabla^2 - \dfrac{k_1+k_2}{k_1}\right]\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} W_{mn}(x,y)} \tag{13}$$

The differential equations for the temporal functions may be obtained by multiplying both sides of Equation 13 by the eigen function and integrating over the plate region $0 \le x \le a$, $0 \le y \le b$. Thus one obtains ordinary differential equations for $T_{mn}(t)$ in the form

$$\ddot{T}_{mn}(t) + 2\xi\omega_{mn}\dot{T}_{mn}(t) + \left(\omega_{mn}\right)^2 T_{mn}(t) = -\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \frac{k_1\int_0^a\int_0^b W_{mn}(x,y)\left[\dfrac{G}{k_1}\nabla^2 - \dfrac{k_1+k_2}{k_1}\right]p(x,y,t)dxdy}{\rho h Q_{mn} G\left[\dfrac{m^2\pi^2}{a^2} + \dfrac{n^2\pi^2}{b^2}\right] + k_1 + k_2} \tag{14}$$

where Q_{mn} is the normalization factor.

The total solution of Equation 14 are

$$T_{mn}(t) = \hat{T}_{mn}(t) + T^*{}_{mn}(t) \tag{15}$$

The particular solutions $T^*_{mn}(t)$ can be represented in the form of Duhamel integrals [6] as follows

$$T^*_{mn}(t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\int_0^t \frac{k_1 e^{-\xi\omega_{mn}(t-\tau)}\sin\sqrt{(1-\xi^2)}\omega_{mn}(t-\tau)}{\rho h Q_{mn}\sqrt{(1-\xi^2)}\omega_{mn}\left[G\nabla^2 W_{mn}(x,y) - (k_1+k_2)W_{mn}(x,y)\right]}$$
$$\int_0^a\int_0^b\left[\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\left[\frac{G}{k_1}\nabla^2 - \frac{k_1+k_2}{k_1}\right]p(x,y,t)dxdy\right]d\tau \tag{16}$$

5. Load Function

The general forced response of orthotropic damped rectangular plates due to an arbitrary surface load and biaxial in-plane loads is given by Equation 9 where the function $T_{mn}(t)$ are expressed by Equation 15.

In order to presenting useful design results, the particular solutions of Equation 16 is presented in the following two cases. For convenience homogeneous initial conditions are assumed for these two cases.

5.1. Concentrated Harmonic Force

The loading function for a concentrated harmonic force of magnitude $P_0\cos\omega t$ applied at (x_0,y_0) is represented by

$$p(x,y,t) = P_0 \cos\omega t\,\delta[x - x_0]\,\delta[y - y_0] \tag{17}$$

where δ is the Dirac-Delta function.

Accordingly, the total dynamic response at any point (x,y) at time t is given by

$$\begin{aligned} w(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\frac{P_0\sin\frac{m\pi x_0}{a}\sin\frac{n\pi y_0}{b}}{\rho h Q_{mn}\sqrt{(1-\xi^2}\,\omega_{mn}} \\ \left[\frac{1}{2}\left[\frac{(f_1-f_3)\cos(f_3t)}{f_4^{\,2}+(f_3-f_1)^2}+\frac{(f_1+f_3)\cos(f_3t)}{f_4^{\,2}+(f_3+f_1)^2}\right]+\frac{1}{2}\left[\frac{f_4\sin(f_3t)}{f_4^{\,2}+(f_3-f_1)^2}-\frac{f_4\sin(f_3t)}{f_4^{\,2}+(f_3+f_1)^2}\right]\right. \\ \left.-\left[\frac{e^{-f_4t}}{2}\left[\frac{(f_1-f_3)\cos(f_1t)}{f_4^{\,2}+(f_3-f_1)^2}+\frac{(f_1+f_3)\cos(f_1t)}{f_4^{\,2}+(f_3+f_1)^2}\right]+\frac{e^{-f_4t}}{2}\left[\frac{f_4\sin(f_1t)}{f_4^{\,2}+(f_3-f_1)^2}+\frac{f_4\sin(f_1t)}{f_4^{\,2}+(f_3+f_1)^2}\right]\right]\right] \end{aligned} \tag{18}$$

where $f_1 = \sqrt{1-\xi^2}\,\omega_{mn}, f_2 = \frac{nv\pi}{b}, f_3 = \omega, f_4 = \xi\omega_{mn}$.

5.2. Load Moving in y Direction

In order to demonstrate the use of this analysis for a moving load, consider a load $P_0\cos\omega t$ that travels at constant velocity v along a straight line path of distance x_0 about the edge of the plate. The loading function p(x,y,t) for this case is written

$$p(x,y,t) = P_0 \cos\omega t\,\delta[x - x_0]\,\delta[y - vt] \tag{19}$$

where P_0, ω, x_0 dan v are constants.

Substituting Equation 19 into Equation 16 and is subsequently into Equation 9 one obtains the total dynamic response at any point (x,y) at time t as follows

$0 \le t \le t_0$:

$$\begin{aligned} w_{mn}(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\left[e^{-f_4t}\sin\left[\frac{m\pi x}{a}\right]\sin\left[\frac{n\pi y}{b}\right]\right]\left[\left(a_{mn}e^{if_1t}+b_{mn}e^{-if_1t}\right)\right. \\ +\int_0^t \frac{k_1 e^{-f_4(t-\tau)}\sin f_1(t-\tau)}{\rho h Q_{mn} f_1\left[G\nabla^2 W_{mn}(x,y)-(k_1+k_2)W_{mn}(x,y)\right]} \\ \left.\int_0^a\int_0^b \sin\left[\frac{m\pi x}{a}\right]\sin\left[\frac{n\pi y}{b}\right]\left[\frac{G\nabla^2}{k_1}-\frac{k_1+k_2}{k_1}\right]P_0\cos\omega\tau\,\delta[x-x_0]\,\delta[y-v\tau]\,d\tau\right]\Bigg] \end{aligned} \tag{20}$$

$t \geq t_0$

$$w(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} e^{-f_4(t-t_0)}\left[w_{0mn} f_1 \cos\left[f_1\left(t-t_0\right)\right] + \frac{v_{0mn} + f_4 w_{0mn}}{f_1}\sin\left[f_1\left(t-t_0\right)\right]\right] \tag{21}$$

where t_0 is the time at which the load starts to leave the plate region, w_{0mn} is the initial deflection and v_{0mn} is the initial velocity.

6. Numerical Results

Numerical results are obtained for a 0.00216m-thick plate of an orthotropic material. The foundation parameters are taken as λ_1=1, λ_2=20, and μ=100. When a rectangular orthotropic plate rests on a Kerr foundation and the plate is compressed by uniformly distributed in-plane loads N_x and N_y, in the instability condition the critical in-plane load is $\overline{N} = 2.112434$ as shown in Figure 3.

In Table 1 are presented the calculated results of the frequency coefficient $\overline{\omega} = (a/\pi)^2 \omega(\rho h / H)^{0.5}$ for the first fifteen frequencies for the square plate supported by the Winkler, the Pasternak and the Kerr foundation model.

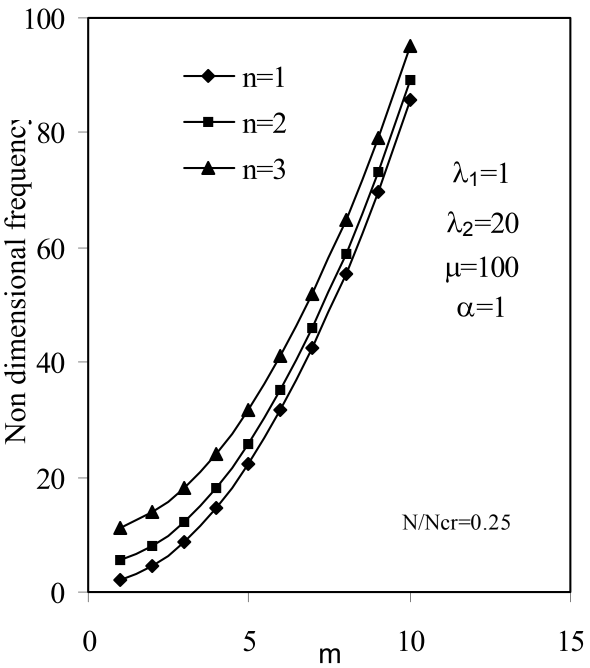

Fig 2. Frequency parameters of the plate with all edges simply supported.

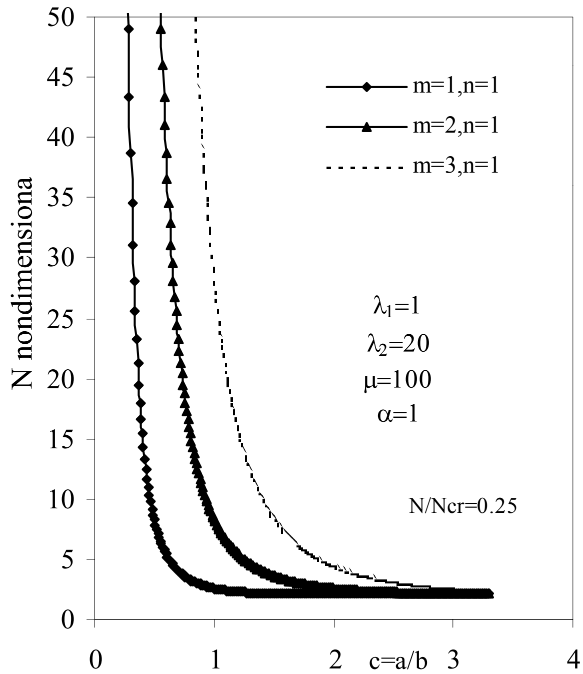

Fig 3. Plate with all edges simply supported and compressed uniformly in both x and y directions.

Table 1 Frequency parameter ($\bar{\omega} = \left(a/\pi\right)^2 \omega\left(\rho h/H\right)^{0.5a}$)

Number	m	n	Winkler λ=19	Pasternak λ=15, μ=1	Kerr λ_1=1,λ_2=20, μ=100
1	1	1	2,00979863	2,00979598	2,22833386
2	2	1	4,55372563	4,55371831	4,65432287
3	3	1	8,780164	8,78015791	8,83275566
4	4	1	14,6932942	14,6932881	14,7247811
5	5	1	22,2945301	22,2945232	22,3152941
6	1	2	5,470982	5,470981	5,554982
7	2	2	8,03921	8,039208	8,096608
8	3	2	12,28755	12,28754	12,32517
9	4	2	18,21498	18,21498	18,24038
10	5	2	25,82516	25,82516	25,84308
11	1	3	11,23084	11,23084	11,272
12	2	3	13,81409	13,81408	13,84757
13	3	3	18,08823	18,08821	18,11381
14	4	3	24,04133	24,04132	24,06058
15	5	3	31,67237	31,67235	31,68699

[a]D_x=35.2026Nm, D_{xy}=14.5893Nm; a=b=0.1m, D_y=65.4616N.m; D_{66}=17.3863Nm, h=0.00216m, ρ=1650kg/m^3, α=1, N/N_{cr}=0.25.

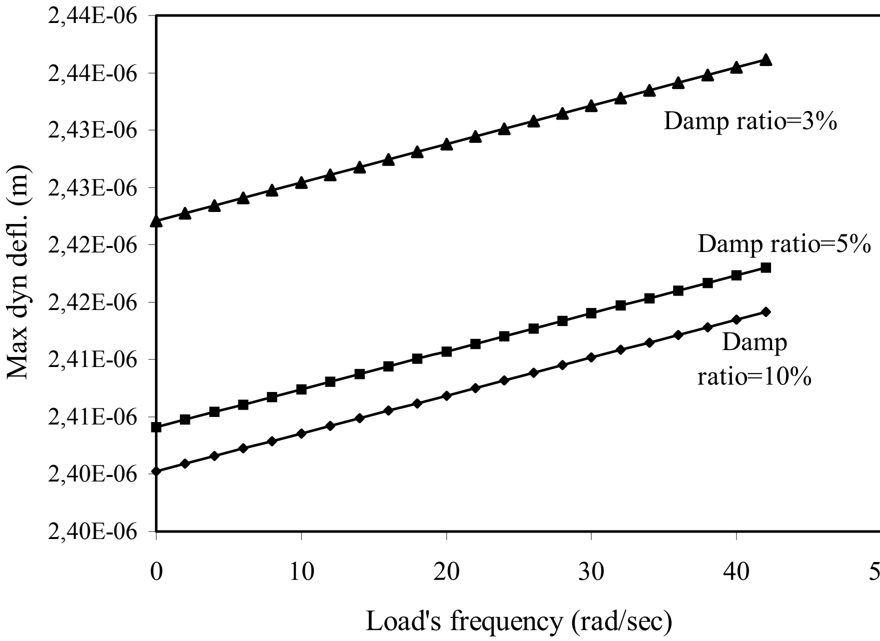

Figure 4. Total maximum deflection response spectra of orthotropic plate on a Kerr foundation subjected to the dynamic moving load.

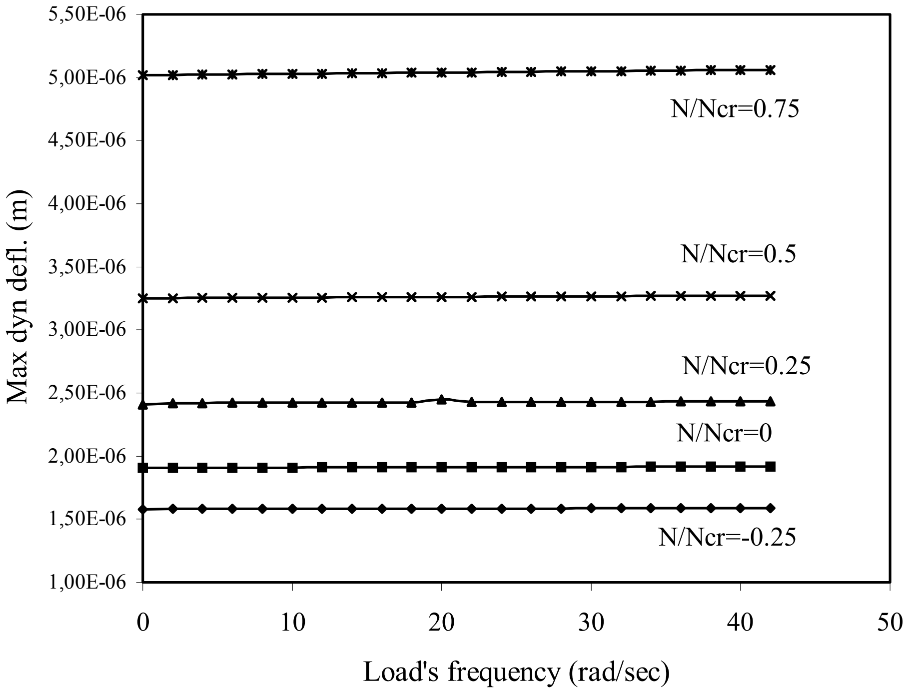

Figure 5. Total dynamic deflection response spectra an orthotropic plate on a Kerr foundation subjected to the dynamic moving load.

Figure 4 shows the maximum dynamic deflection response spectra of an orthotropic plate on a Kerr foundation model subjected to a concentrated dynamic moving load P_0=1000N, which travels with a constant velocity v=0.02m/sec for various damping ratio. Figure 5 shows the maximum dynamic deflection response spectra of an orthotropic plate on a Kerr foundation model as subjected to a concentrated dynamic moving load and biaxial in-plane forces. It is shown that the maximum dynamic deflection increases as the ratio between N to N_{cr} increases. Figure 6 shows the dynamic deflection mode shapes subjected to the dynamic moving load for $0 \le t \le t_0$.

7. Conclusion

The analysis presented herein thus provide a unified approach for determining the stability and the forced response of elastically supported orthotropic plates subjected to an arbitrary surface load p(x,y,t). The general solution for forced response is presented in such a way that it may be readily applied to a wide variety of engineering design situations.

The foundation parameters μ, λ_1, λ_2 and the ratio between N to N_{cr} play an important role in the dynamic characteristics of orthotropic plate.

The dynamic deflection is a function of the damping ratio and the ratio between N to N_{cr}. As expected, the total dynamic deflection decreases as the damping increases. Damping plays its usual role of reducing the maximum dynamic deflection. Analysis of this nature is particularly useful for applications that have constraints on the allowable maximum dynamic deflection.

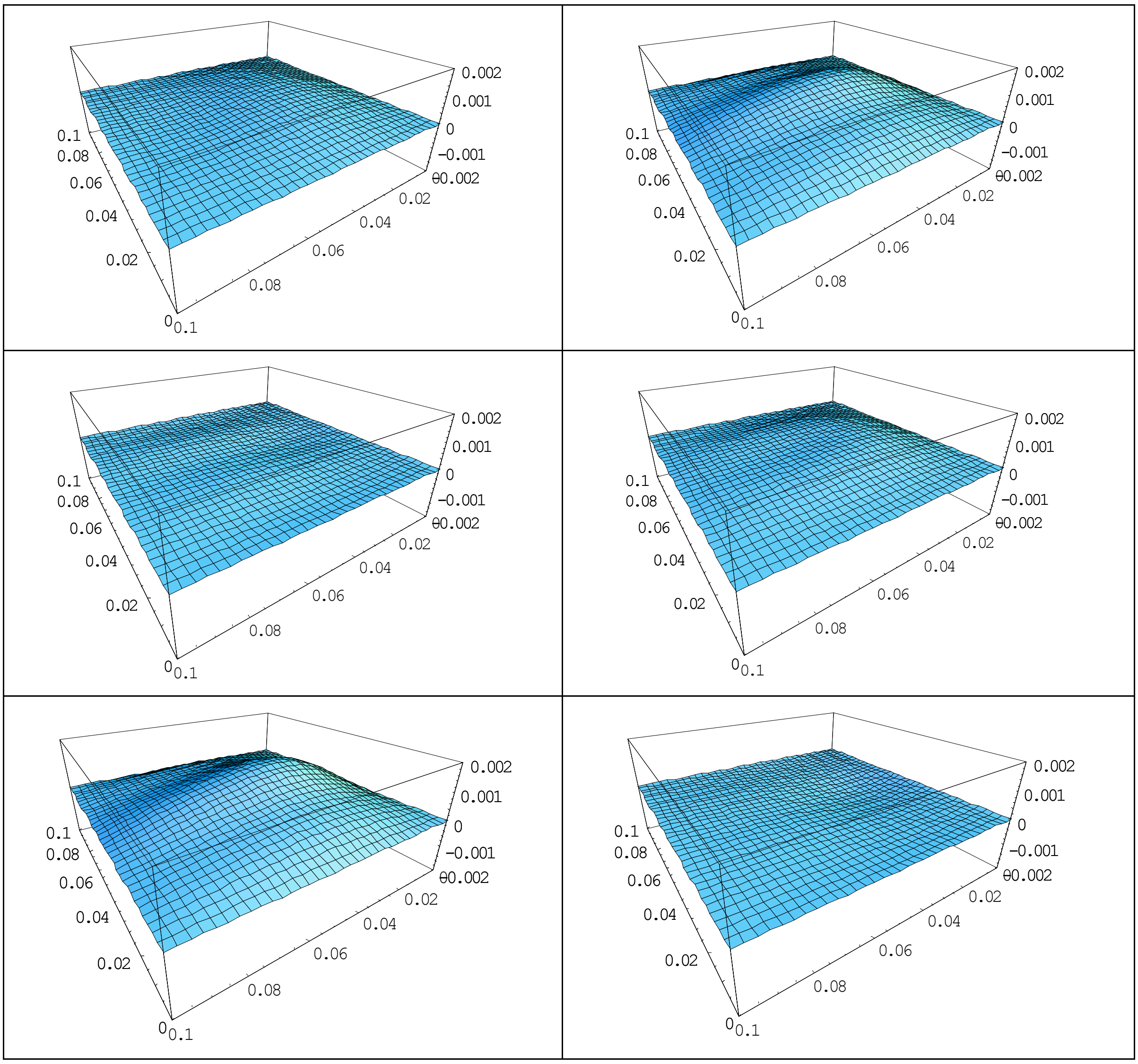

Figure 6. The mode shapes of a plate simply supported on all sides due to dynamic moving load.

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