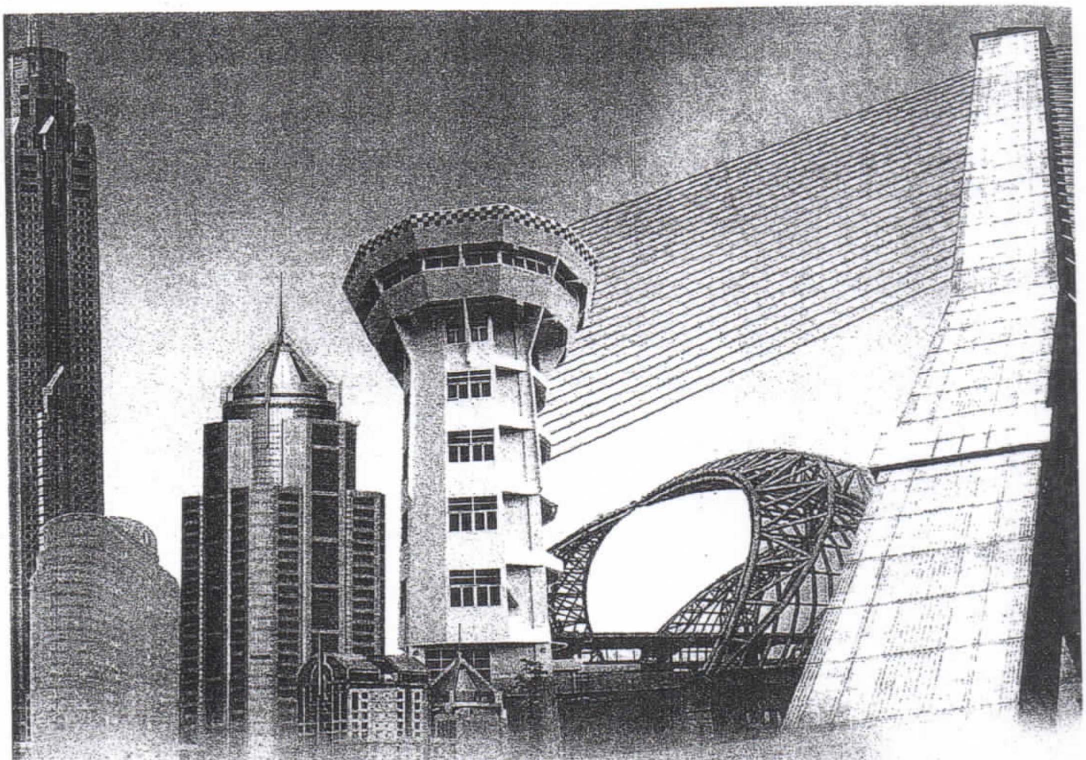


WIND AND EARTHQUAKE ENGINEERING

PROCEEDINGS OF

The Tenth East Asia-Pacific Conference on Structural Engineering & Construction (EASEC-10)

Bangkok, Thailand

EASEC
THE EAST ASIA-PACIFIC CONFERENCE ON STRUCTURAL ENGINEERING & CONSTRUCTION
20th ANNIVERSARY

Edited by
Worsak Kanok-Nukulchai
Sunil Munasinghe
Naveed Anwar

The Tenth East Asia-Pacific Conference on Structural Engineering and Construction
August 3-5, 2006, Bangkok, Thailand

RESPONSE DYNAMICS OF RIGID RUNWAY PAVEMENTS

Sofia W. ALISJAHBANA[1], Wiratman WANGSADINATA[2]

ABSTRACT: In this paper response dynamics of rigid runway pavements subjected to moving dynamic loads are investigated. To solve this problem, the rigid runway pavement is modeled as a damped rectangular orthotropic plate resting on a Pasternak foundation. This type of elastic foundation model is introduced to accommodate shear reactions between the spring elements. The rigid runway pavement's natural frequencies are presented in a form analogous to those of a simply supported plate as wave numbers. These wave numbers are determined from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method. The dynamic loading function is described as a traveling concentrated load of harmonically varying amplitude moving suddenly to a new position at $t=t_0$ and continues to travel with a constant speed. The response dynamics of rigid runway pavement are expressed in integral form that is readily to be solved using the Duhamel integration method. The results determine the forced responses of the runway slab under the action of the moving wheel load of the aircraft during landing.

KEYWORDS: rigid runway pavement, Pasternak foundation, Modified Bolotin Method

1. INTRODUCTION

The study of the dynamic responses of a plate resting on an elastic foundation such as the Pasternak foundation subjected to moving loads is important, as the results may contribute to the understanding of the dynamic behavior of runway and roadway pavements. In the analysis of runway pavements of airports, the structure is usually modeled as an orthotropic plate resting on an elastic foundation. In general, loads on these types of structures are moving loads such as the wheel loads from moving planes during take off and landing. Static and free vibration analyses of plates resting on an elastic foundation had been studied extensively, for example by Saha [1], Matsunaga [2] and Pevzner et al. [3]. Dynamic response of plates resting on an elastic foundation has attracted much less attention. Gbadeyan and Oni [4] gave a closed form solution by using a double Fourier sine integral transformation to analyze a simply supported rectangular plate resting on an elastic Pasternak foundation subjected to an arbitrary number of moving concentrated masses. Huang and Thambiratnam [5] had investigated the dynamic response of plates on elastic foundation subjected to moving loads by using the finite strip method and a spring system. In the numerical analysis, the Wilson-θ method is adopted for direct integration. It is important to note that in most actual structures, the support conditions of the plates are both complex and irregular as required by the engineering designs. Runway pavements of military aircrafts are often subjected to high levels of acoustic pressure. Therefore, the runway pavement may vibrate with large amplitude displacements, i.e., with geometrical nonlinearity. Such nonlinearity may cause multi-modal interaction and lead to internal resonance. One of the most efficient methods to avoid damage due to internal resonance is to

[1] *Professor and Director School of Graduate Studies, Tarumanagara University, Indonesia*
[2] *Professor Emeritus, Tarumanagara University, Indonesia*

introduce irregular internal supports [6]. The problems of plates with an irregular support condition are far more complicated to be numerically solved. What is the most relevant to the present work is the use of the Modified Bolotin Method for solving the vibration modes of rectangular plates [3]. Previous extensive studies of the dynamic response of plates supported by an elastic foundation with simply supported and unsymmetrical boundary conditions had been investigated by Alisjahbana and Wangsadinata [7-8].

One objective of this paper is to call further attention to the dynamics of runway pavements subjected to aircraft loadings, modeled as orthotropic rectangular plates supported by a Pasternak foundation with unsymmetrical boundary conditions. The solution of the problem is illustrated by the results of a case study. Another objective is to demonstrate the application of the Modified Bolotin Method in handling this class of problems. Results of dynamic response analyses are presented and the effects of harmonic load frequency, moment distribution, shear distribution and response spectra of the system are further shown.

2. THE PROBLEM AND METHODS OF SOLUTION

The problem of the dynamic response of a runway pavement modeled as an orthotropic rectangular plate supported by a Pasternak foundation with unsymmetrical boundary conditions is described. The Modified Bolotin Method is only briefly described in this section, so that the reader should refer to the original work for more detailed information [3].

2.1. PLATE VIBRATION

Although the attention is limited to the vibration of classical rectangular Kirchhoff plates with simply and transversely supported edges, the method can be applied to many other applications in solid mechanics. Let us consider a rectangular orthotropic plate supported by a Pasternak foundation of length a, width b, thickness h, mass density ρ, flexural rigidity in x and y direction respectively D_x and D_y, torsional rigidity B, foundation's spring stiffness k_f, foundation's shear modulus G_s, Poisson's ratio in x and y direction respectively υ_x and υ_y. The origin of the Cartesian coordinates (x,y) is set at the upper left corner of the plate. The governing differential equation for the orthotropic plate is given by

$$D_x\frac{\partial^4 w}{\partial x^4}+2B\frac{\partial^4 w}{\partial x^2\partial y^2}+D_y\frac{\partial^4 w}{\partial y^4}-\rho h\omega^2 w+k_f w-G_s\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right]=0 \qquad (1)$$

where $w(x,y)$ is the transverse deflection of the mid surface of the plate and ω is the circular frequency. The two considered types of support conditions for each plate edge are as follow:

For simply supported edge

$$-D_y\left(\frac{\partial^2 w}{\partial y^2}+\nu_x\frac{\partial^2 w}{\partial x^2}\right)=0;\quad w=0 \text{ along y=0 and y=b} \qquad (2)$$

For transversely supported edge with non uniform elastic rotational restraint

$$-D_x\left(\frac{\partial^2 w}{\partial x^2}+\nu_y\frac{\partial^2 w}{\partial y^2}\right)=k_1\frac{\partial w}{\partial x};\quad w=0 \text{ along x=0} \qquad (3)$$

$$-D_x\left(\frac{\partial^2 w}{\partial x^2}+\nu_y\frac{\partial^2 w}{\partial y^2}\right)=k_2\frac{\partial w}{\partial x};\quad w=0 \text{ along x=a} \qquad (4)$$

where k_1 and k_2 are the elastic rotational stiffness constants of the plate's edge.

2.2. THE MODIFIED BOLOTIN METHOD (MBM)

In the MBM an eigen mode is initially approximated by a general solution consisting of trigonometric functions (in the interior region)

$$w = A_{mn} \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi y}{b}\right) \tag{5}$$

where p and q are real numbers which are to be determined. From Equations (5) and (1) the formula for the eigen frequency derived by the MBM is given by

$$\omega_{mn}{}^2 = \left(\frac{\pi^4}{\rho h}\right)\left[D_x\left(\frac{p}{a}\right)^4 + 2B\left(\frac{pq}{ab}\right)^2 + D_y\left(\frac{q}{b}\right)^4\right] + \frac{k_f}{\rho h} + \frac{G_s}{\rho h}\left[\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2\right] \tag{6}$$

The solution of the first auxiliary problem satisfying the boundary conditions according to Equations (3) and (4) can be expressed as

$$X(x) = A_1 cosh\left[\frac{\beta\pi x}{ab}\right] + A_2 sinh\left[\frac{\beta\pi x}{ab}\right] + A_3 cos\left[\frac{p\pi x}{a}\right] + A_4 \sin\left[\frac{p\pi x}{a}\right] \tag{7}$$

where

$$\beta = \sqrt{\left[\frac{2q^2b^2B}{D_x} + p^2b^2 + \frac{G_s a^2 b^2}{\pi^2 D_x}\right]} \tag{8}$$

Substituting of Equation (7) into the boundary conditions according to Equations (3) and (4), the existence of a nontrivial solution yields the first characteristic determinant.

The solution of the second auxiliary problem satisfying the boundary conditions according to Equation (2) can be expressed as

$$Y(y) = B_1 \sin\left[\frac{q\pi y}{b}\right] \tag{9}$$

2.3. DYNAMIC RESPONSES OF THE PLATE

The dynamic response of the plate is determined by using the method of variation of parameters, which can be written in the following form:

$$\begin{aligned} w(x,y,t) = \Bigg[& \cosh\left[\frac{\beta\pi x}{ab}\right] + \frac{b\left[c_1 k_1 p - C_1 k_1 p + a\left(F_1 + F_2\right)s_1\right]}{k_1 p S_1 + s_1 \beta} \sinh\left[\frac{\beta\pi x}{ab}\right] \\ & \sin\left[\frac{p\pi x}{a}\right] - \cos\left[\frac{p\pi x}{a}\right] + \left[\frac{ab\left(F_1 + F_2\right)S_1 + \left(c_1 - C_1\right)k_1\beta}{-bpS_1 + s_1\beta}\right] \sin\left[\frac{p\pi x}{a}\right]\Bigg] \sin\left[\frac{q\pi y}{b}\right] T_{mn}(t) \end{aligned} \tag{10}$$

where T_{mn} (t) is a function of time, which must be determined through further analysis.

The general solution for the forced response deflection of the plate to an arbitrary dynamic load p(x,y,t) is given in integral form by using Duhamel's integral method as follows:

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[X_m(x)Y_n(y)e^{-\gamma\omega_{mn}t}\left[a_{mn}\cos\left[\sqrt{1-\gamma^2}\,\omega_{mn}\right]t+b_{mn}\sin\left[\sqrt{1-\gamma^2}\,\omega_{mn}\right]t\right]\right]+$$
$$+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\int_o^t\left[\frac{p(x,y,\tau)}{\rho h Q_{mn}}\int_o^a X_m(x)dx\int_o^b Y_n(y)dy\right]\left[\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\sqrt{1-\gamma^2}\,\omega_{mn}}\sin\sqrt{1-\gamma^2}\,\omega_{mn}(t-\tau)\right]\right]d\tau \tag{11}$$

3. RESULTS AND DISCUSSION

An orthotropic rectangular plate supported by an elastic Pasternak foundation subjected to a dynamic load is considered. The data for the plate and load amplitude for the numerical example treated in this paper are: a=7.5 m, b=15 m, h=0.5 m, E_x=30x10^9 N/m^2, E_y=20x10^9 N/m^2, υ_x=0.15, υ_y=0.1, ρ=2.4x10^3 kg/m^3, G_p=10^{10} N/m^2 and P_0=6.0x10^5 N. The boundary conditions are transversely supported edges with non uniform elastic rotational restraint along the shorter edges (x=0, 7.5 m) and simply supported along the longitudinal edges (y=0, 15 m). In the following discussion, x_0 and y_0 refer to the moving load position, Δx refers to the distance of the suddenly moving load, t_0 refers to the time at which the load is moving to the new position, t_1 refers to the time at which the load starts to move with a constant velocity v. In this numerical example, the elastic foundation stiffness is set to be k_p=6.25x10^6 N/m^3.The load moves along the centerline (y_0=7.5m) parallel to the x axis with constant amplitude. At time $t=t_0$, the load suddenly moves to a new position at $x=x_1$, and continues moving with a constant velocity v=280 km/hr. Figure 1 shows the dynamic response spectra as a function of the load's frequency ω and damping ratio γ (0, 0.05 and 0.10). It can be seen that the dynamic deflection will increase significantly when the load's frequency approaches the first natural frequency of the orthotropic plate (ω_{11}=220.56 rad/sec). Finally, Figure 2 shows the various responses of the orthotropic plate, where it is apparent that at low damping the load frequency does affect not only the maximum but also the distribution of the various responses.

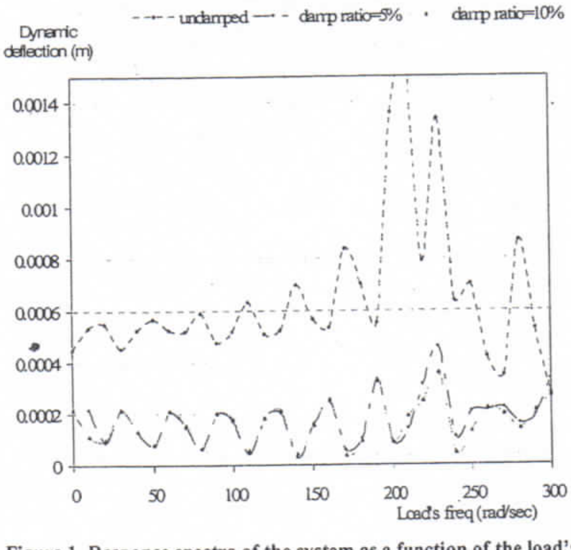
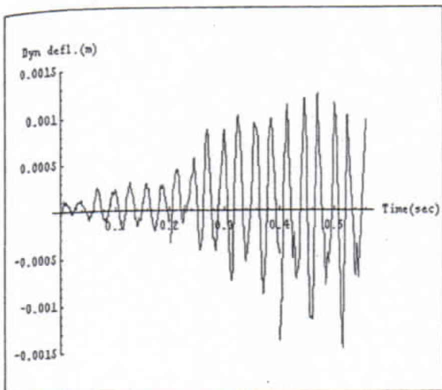
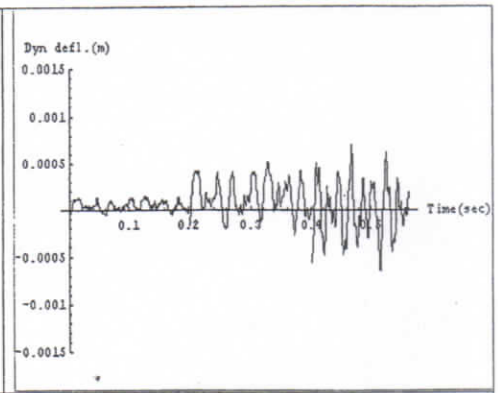


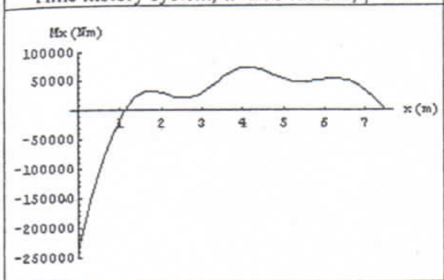
Figure 1. Response spectra of the system as a function of the load's frequency for various values of damping ratio.



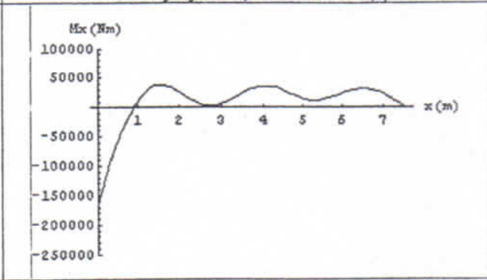
Time history system, ω=200 rad/sec, γ=0%



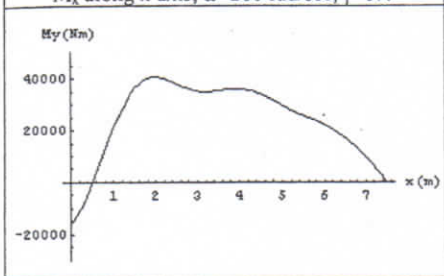
Time history system, ω=50 rad/sec, γ=0%



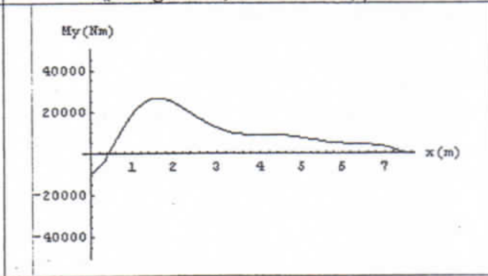
M_x along x axis, ω=200 rad/sec, γ=0%



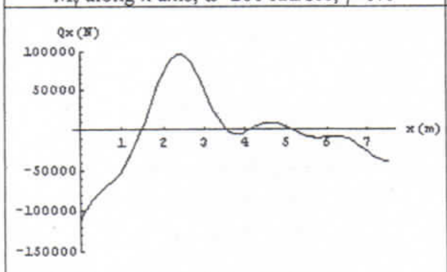
M_x along x axis, ω=50 rad/sec, γ=0%



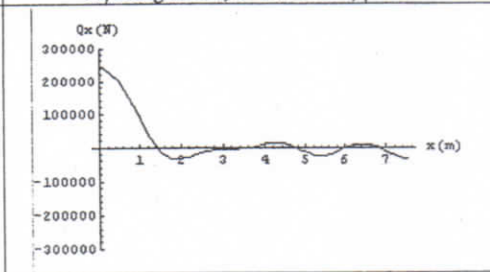
M_y along x axis, ω=200 rad/sec, γ=0%



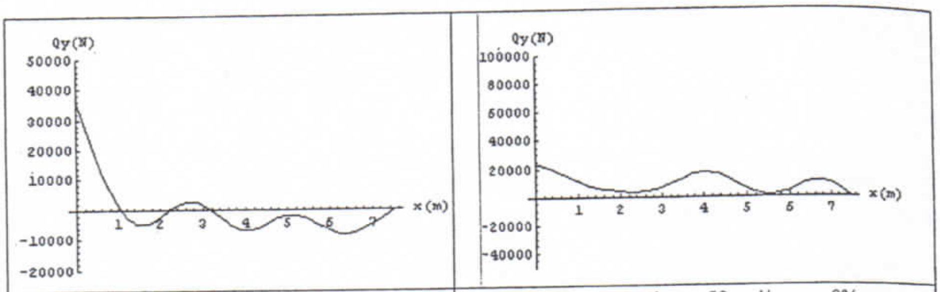
M_y along x axis, ω=50 rad/sec, γ=0%



Q_x along x axis, ω=200 rad/sec, γ=5%



Q_x along x axis, ω=50 rad/sec, γ=0%



Q_y along x axis, ω=200 rad/sec, γ=0%	Q_y along x axis, ω=50 rad/sec, γ=0%

Figure 2. Various dynamic responses of the plate at near resonance condition (left) and away from resonance condition (right).

4. CONCLUSIONS

Based on the above study the following can be concluded:

1) The use of the Modified Bolotin Method (MBM) to approximate natural frequencies and mode shapes of rectangular orthotropic plates with any type of edge restraints, leads to reasonably accurate mode shapes for the entire region of the plate.
2) The theory of the rectangular damped orthotropic plate resting on a Pasternak foundation with any type of edge restraints subjected to moving dynamic loads, can reasonably be applied in the analysis of rigid runway pavements under the action of the moving wheel load of an aircraft during landing.
3) The results of dynamic response analyses provide a better understanding of the orthotropic plate behavior under the effect of the moving dynamic load, indicating possible resonance potentials as well.
4) Through dynamic response analyses one may justify the appropriateness of a selected combination of the orthotropic plate and foundation material properties, since it is the combined material effect, rather then the individual ones that determines the overall performance of the plate.

5. REFERENCES

1. Saha, K.N. "Dynamic stability of a rectangular plate on non-homogeneous Winkler foundation", *Comp. Struct.*, 63(6), 1997, pp. 1213-1222.
2. Matsunaga, H. "Vibration and stability of thick plates on elastic foundation", *Journal of Eng. Mechanics*, 124(9), 2000, pp. 27-34.
3. Pevzner, P. "Further modification of Bolotin method in vibration analysis of rectangular plates", *AIAA Journal*, Vol. 38, No. 9, 2000, pp. 1725-1729.
4. Gbadeyan, J.A., and Oni, S.T. "Dynamic response to moving concentrated masses of elastic plates on a non-Winkler elastic foundation", *J. Sound Vibration*, 154(2), 1992, pp. 343-358.
5. Huang, M.H and Thambiratnam, D.P. "Dynamic response of plates on elastic foundation to moving loads", *Journal of Eng. Mechanics*, 2002, pp. 1016-1022.
6. Zhao, Y.B. et. al. "Plate vibration under irregular internal supports", *Int. J. of Solids and Structures*, Vol 39, 2002, pp. 1361-1383.
7. Alisjahbana, S.W. and Wangsadinata, W. "Dynamics of rigid roadway pavements". *The 4th Int. Conference on Concrete under Severe Conditions (CONSEC'04)*, Seoul, Korea, June 27-30, 2004.
8. Alisjahbana, S.W. and Wangsadinata, W. "Dynamic response of clamped orthotropic plates to dynamic moving loads, *The 3rd Int. Conference on Construction Materials: Performance, Innovations and Structural Implications,* Vancouver, B.C., Canada, August 22-24, 2005.

EASEC-10 Secretariat

Asian Center for Engineering Computations and Software (ACECOMS)
School of Civil Engineering, Asian Institute of Technology (AIT)
P.O. Box 4, KlongLuang, Pathumthani 12120 Thailand
Tel : (662) 524 5539, Fax (662) 524 6059
E-mail : easec10@ait.ac.th
Website : http://www.easec10.net

ACECOMS

ASIAN INSTITUTE OF TECHNOLOGY 1959