

BLAST LOADED STIFFENED ORTHOTROPIC PLATES

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ABSTRACT

This study is concerned with the numerical study of semi rigid orthotropic stiffened plates subjected to the localized blast load. The aim of this research is to determine the dynamic response of the orthotropic plates with different stiffener configurations; thickness of the plate, and the effect of time duration of the blast loading subjected to localized blast load. Numerical solutions for the natural frequencies and mode shapes are obtained by using the Modified Bolotin Method (MBM). Numbers of mode of orthotropic plates are real numbers and solved from the transcendental equations. Special emphasis is focused on the dynamic deflection of mid-point displacements. The results obtained allow getting an insight into the effect of stiffeners configurations and the time duration of the blast load on the dynamic response of the orthotropic plates under localized blast loading and indicate that stiffener configurations and time duration can affect their overall behaviour.

Key words: *blast load, stiffener configuration, Modified Bolotin Method, transcendental equations*

1. INTRODUCTION

The blast response of stiffened plates has been studied in the past due to different accidental or intentional events of the structural components to blast loading. To provide adequate protection against explosions, the design and the construction of public buildings such as school, hospitals and government buildings are receiving renewed attention from structural engineers. Due to the complexity of the problems that involves time dependent finite deformation, high strain rates, and non-linear inelastic material behaviour have motivated various assumption and simplifications of the problems.

In 1989, Schubak et al [1] have presented a simplified rigid-plastic method for modelling beams. Later, this work has extended to one-way and two-ways orthogonally stiffened plates [2]. In this work, response of a one-way stiffened plate under intense loads, and the stiffened plate was treated as a single symmetric beam with the plate acting as large flange. In 2003, Yuen and Nurick [3] had studied the response of the quadrangular plates with different stiffener configuration under blast loading. Both temperature-dependent material properties were included in the numerical modelling. Jacinto et al in 2001, studied the numerical modelling on metallic plates subjected to explosive loads [4]. A linear dynamic analysis of the plate models with ABAQUS was carried out. The results showed that the effect of plasticity, strain-rate and

buckling has to be included in the analysis of the plate subjected to blast loading. Numerical studies of stiffened plates subjected to hydrocarbon explosions and a parametric study of the simplified model of the stiffened plate under different stress state and different stiffener configurations were presented by Pan and Louca in 1999 [5].

In 2011, Alisjahbana, S.W. and Wangsadinata W. [6] carried out more rigorously the behaviour of orthotropic plates under localized blast load. The orthotropic plate was stiffened by orthogonal beams. The effect of the stiffener configurations, and the inclusion of the viscous dampers in the equation of motion were studied in this work numerically. The blast load was modelled as a step linear triangular function that was numerically solved by using the Duhamel integration method. The eigen values of the problem were solved numerically by using the Modified Bolotin Method (MBM) that significantly influence the response of the orthotropic plates subjected to blast loading.

The main conclusions of these studies are that the modelling of the blast load and the boundary conditions at the edges of the plate has a significant influence on the response of the plate.

2. DESCRIPTION OF THE PLATE

The dynamic responses of the orthotropic plate under blast conditions are highly dependent on the material properties of the concrete and reinforcement of the stiffeners. In this numerical analysis, the assumption that the bonding between the reinforcing bars and the concrete were assumed to be perfect.

The plate thickness of plates are varied from 100 mm to 200 mm thick and $5,500 \times 4,750 \text{ mm}^2$ with rectangular stiffeners parallel to x axes. In Figure1 is shown the rectangular orthotropic stiffened plate with semi rigid boundary conditions along its edges subjected to dynamic load $p(x,y,t)$.

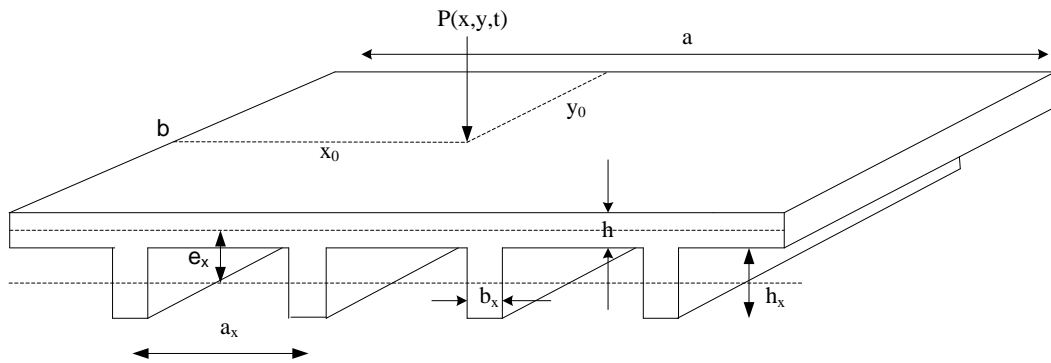


Figure 1. Rectangular orthotropic stiffened plate subjected to dynamic load $p(x,y,t)$.

Using the classical theory of thin plates, the equation of equilibrium of an elastic orthotropic stiffened plate can be expressed as:

$$D_x \frac{\partial^4 w(x, y, t)}{\partial x^4} + 2B \frac{\partial^4 w(x, y, t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w(x, y, t)}{\partial y^4} + \gamma h \frac{\partial w(x, y, t)}{\partial t} + \rho h \frac{\partial^2 w(x, y, t)}{\partial t^2} = p(x, y, t) \quad (1)$$

where D_x and D_y are the flexural rigidity in x and y direction respectively, B is the torsional rigidity, γ is the damping ratio, ρ is the mass density of the plate and $p(x,y,t)$ is the blast load.

3. IDEALISATION OF BLAST LOADING

Simple methods of structural dynamics were applied by Biggs [7] and Clough and Penzien [8] by applying single degree of freedom system (SDOF) and idealizing the plate as the beam for obtaining the blast damage. In this work the blast loading is simplified by a linearly decaying pressure-time history as follows:

$$P(t) = P_{\max} \left(1 - \frac{t}{t_d} \right) \quad (2)$$

where P_{\max} is the maximum amplitude of the blast load, t_d is the time duration of the blast loading that can be expressed by the following equation:

$$p(x, y, t) = P(t)\delta[x - x(t)]\delta[y - y(t)] = P(t)\delta[x - x_0]\delta[y - y_0] \quad (3)$$

where $\delta[.]$ is Dirac delta function, x_0 is the position of the localized blast load in the x direction and y_0 is the position of the localized blast load in the y direction.

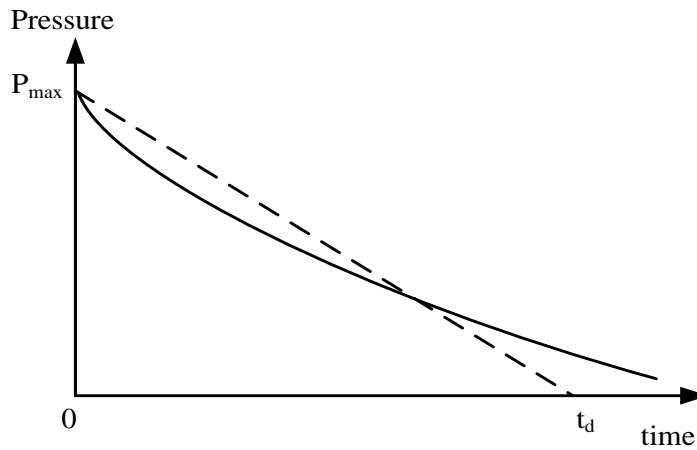


Figure 2. Pressure time history from a blast load – triangular form.

4. DYNAMIC ANALYSIS

The dynamic analysis of the system has been solved by using the method of separation of variable method. The free vibration of the orthotropic stiffened plate with semi rigid condition along its support can be expressed as:

$$w(x, y, t) = W(x, y) \sin \omega t \quad (4)$$

where $W(x,y)$ is a function of the position coordinates only, and ω is the circular frequencies of the system. The circular frequencies of the system for the semi rigid boundary conditions along the edges of the plate can be solved from two transcendental equations obtained from the solution of two auxiliary Levy's type problem as [9]:

$$\omega_{mn} = \sqrt{\left(\frac{\pi^4}{\rho h}\right) \left[D_x \left(\frac{p}{a}\right)^4 + 2B \left(\frac{pq}{ab}\right)^2 + D_y \left(\frac{q}{b}\right)^4 \right]} \quad (5)$$

In eqn. (5) p and q are dynamic modes of the system which are real numbers.

A modal analysis has been conducted to obtain the natural frequencies of the plates for determining the stiffeners effects and the duration of the loading over the natural period of the plate. The general solution for the forced response deflection of the orthotropic stiffened plate to a localized blast load $p(x,y,t)$ is given in integral form as follows:

$$w(x, y, t) = \sum_{m=1}^m \sum_{n=1}^n X_m(x) Y_n(y) \int_0^t \left[\frac{P(\tau) \delta[x - x_0] \delta[y - y_0]}{\rho h Q_{mn}} \int_0^a X_m(x) dx \int_0^b Y_n(y) dy \right] \frac{e^{-\gamma \omega_{mn}(t-\tau)}}{\omega_{mn} \sqrt{1-\gamma^2}} \sin \omega_{mn}(t-\tau) d\tau \quad (6)$$

The circular frequencies of the system for the first 5 modes in the x direction ($m=1,2,\dots, 5$) and the first 5 modes in the y directions ($n=1,2,\dots, 5$) are shown in Table I for different stiffeners configurations.

Table I: The circular frequencies of the system for the first 5 modes in the x direction ($m=1,2,\dots, 5$) and the first 5 modes in the y directions ($n=1,2,\dots, 5$).

m	n	t=0.10m	t=0.12m	t=0.14m	t=0.16m	t=0.18m	t=0.20m
		ω_{mn} (rad/s)	ω_{mn} (rad/s)	ω_{mn} (rad/s)	ω_{mn} (rad/s)	ω_{mn} (rad/s)	ω_{mn} (rad/s)
1	1	148.251	169.867	192.269	215.374	235.545	263.172
	2	348.911	404.346	462.745	522.938	580.903	646.777
	3	636.952	747.495	861.909	978.662	1093.47	1217.27
	4	1016.69	1201.7	1391.31	1583.72	1773.87	1975.9
	5	1488.97	1767.93	2051.76	2338.93	2622.91	2923.87
2	1	319.177	338.605	367.304	398.86	428.117	467.795
	2	520.01	572.575	633.545	699.947	762.674	842.914
	3	818.376	920.951	1035.39	1157.14	1275.48	1414.38
	4	1206.25	1380.06	1568.17	1764.96	1958.54	2175.76
	5	1684.86	1950.18	2231.56	2522.76	2810.4	3126.48
3	1	610.632	646.899	691.595	742.21	793.79	855.227
	2	813.649	871.813	944.083	1025.84	1105.92	1207.6
	3	1119.21	1228.99	1341.11	1475.27	1606.84	1768.3
	4	1516.87	1682.41	1873.53	2080.74	2285.28	2525.88
	5	2004.6	2256.46	2538.24	2838.57	3136.06	3475.96
4	1	1031.16	1086.78	1156.67	1236.47	1320.9	1415.69
	2	1230.45	1303.72	1397.51	1505.32	1615.31	1748.27
	3	1539.61	1648.65	1786.19	1942.72	2099.83	2292.32
	4	1945.35	2111.82	2314.41	2540.78	2767.09	3039.25
	5	2443.59	2688.66	2977.65	3294.66	3611.26	3983.31
5	1	1573.56	1655.4	1758.84	1877.23	2004.34	2143.49
	2	1769.71	1866.4	1991.3	2135.5	2286.55	2461.82
	3	2080.24	2207.34	2371.69	2561.26	2756.06	2989.61
	4	2492.76	2669.62	2893.88	3149.67	3409.91	3722.64
	5	2999.89	3247.94	3553.47	3896.59	4243.37	4656.46

5. RESULTS AND DISCUSSIONS

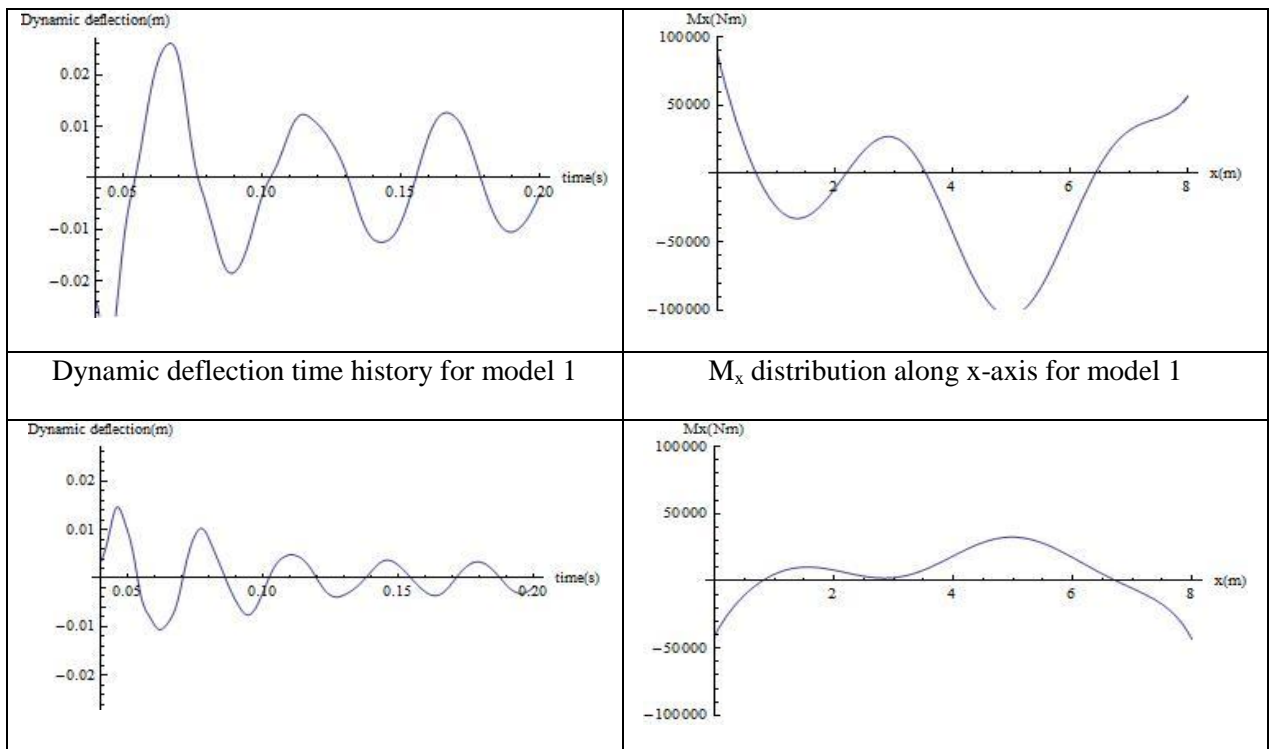
In order to obtain a general view of the dynamic responses of the orthotropic stiffened plates under a localized blast loading, six various thickness of the plates and three stiffeners configurations are considered in this paper. The data for the orthotropic floor plate and blast load are: $a=5.5$ m, $b=4.75$ m, $E_c=2.57E^9$ N/m², $\rho=2400$ kg/m³, $P_{max}=1.3E^6$ N/m, $t_d=1$ ms, 2 ms and 20 ms, $x_0=1/3a$, $y_0=1/3b$.

5.1. Effect of Stiffeners Configurations

The absolute maximum dynamic deflection has been computed for 3 different stiffeners configuration. The introduction of stiffeners decreases the mid-point displacement significantly; the mid-point displacement for model 1 (without stiffener) is 7.87 mm, while for model 2 (1 stiffener) and model 3 (2 stiffeners) are 4.41 mm and 3.40 mm respectively, all computed for the value of damping ratio of 5%. Thus, the configurations of stiffeners can have an important influence on the response of the stiffened orthotropic plates as shown in Table II.

Table II: The absolute maximum dynamic deflection of a damped orthotropic stiffened plate subjected to a triangular load for different values of damping ratio and stiffeners configuration.

Damping ratio (%)	Model 1 (without stiffener)	Model 2 (1 stiffener)	Model 3 (2 stiffeners)
	w_{max} (mm)	w_{max} (mm)	w_{max} (mm)
1	14.45	10.17	9.51
3	10.45	6.47	5.21
5	7.87	4.41	3.40
10	4.43	2.11	1.79



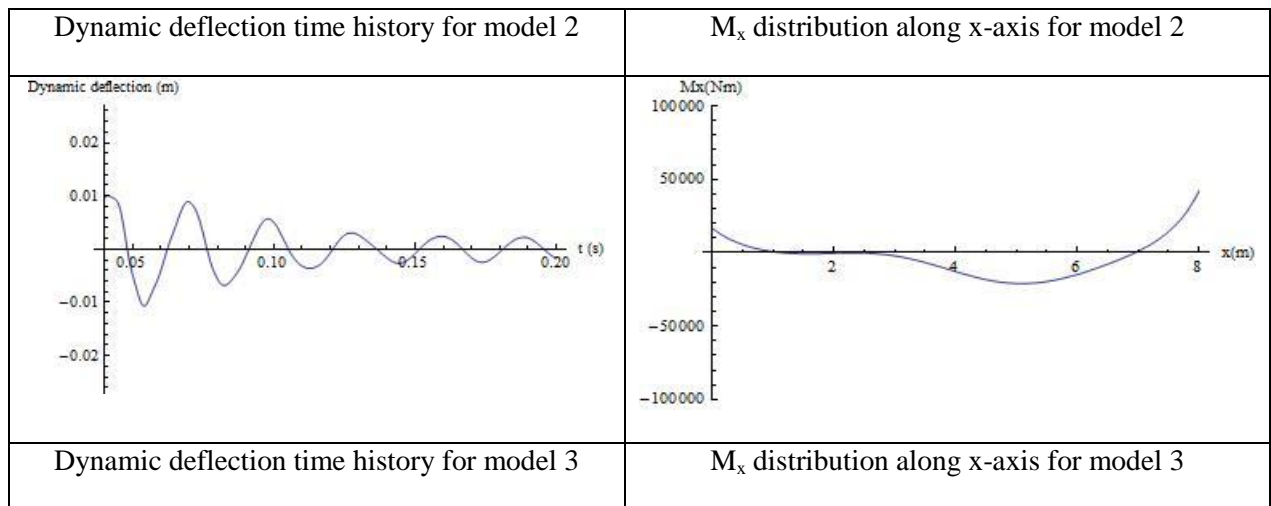
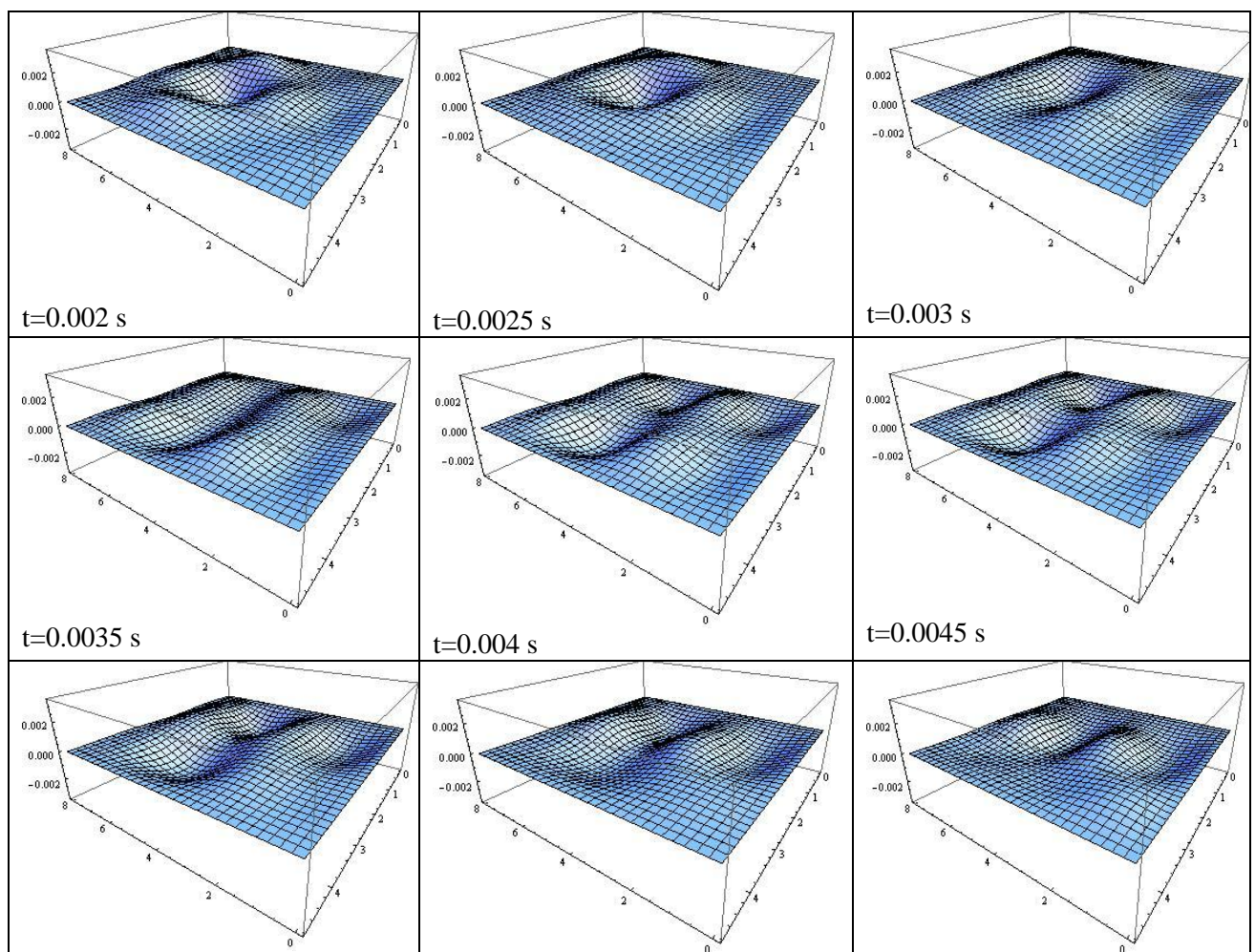


Figure 3: Response of damped orthotropic plate for model 1, model 2 and model 3 subjected to a triangular blast load.

In Figure 3 the mid-point displacement time histories and the x-moment (M_x) distribution along the x-axes are shown for damping ratio equal to 5% and $t_d=2$ ms for model 1, model 2 and model 3. It can be seen, that the dynamic deflection of the mid-point and the x-moment distribution along x-axes of model 2 and model 3 are relatively smaller than that of model 1 due to the greater stiffeners effect. It can also be seen, that the mid-point deflection are decaying with time due to the damping effect. The deformation of the stiffened orthotropic plate at different times is shown in Figure 4.



t=0.005 s	t=0.0055 s	t=0.006 s
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Figure 4: The deformation of the damped orthotropic plate for model 3 (2 stiffeners) with the value of damping ratio 5% subjected to a triangular blast load, $t_d= 2\text{ms}$.

5.2. Effect of the Plate Thickness

For model 2 (1 stiffener) increasing the floor thickness from 0.16 m to 0.18 m for damped system ($\gamma=5\%$) has resulted in a decrease in the mid-point displacement by 32.6 %. While, for model 3 (2 stiffeners) increasing the floor thickness from 0.16 m to 0.18 m for damped system ($\gamma=5\%$) has resulted in a decrease in the mid-point displacement by 32.4 % as shown in Figure 5. Increasing the thickness of the floor has also resulted in a decrease of the distribution of the internal moment in the x direction (M_x) along the x axes for both damped system $\gamma=5\%$ and $\gamma=10\%$, as shown in Figure 6.

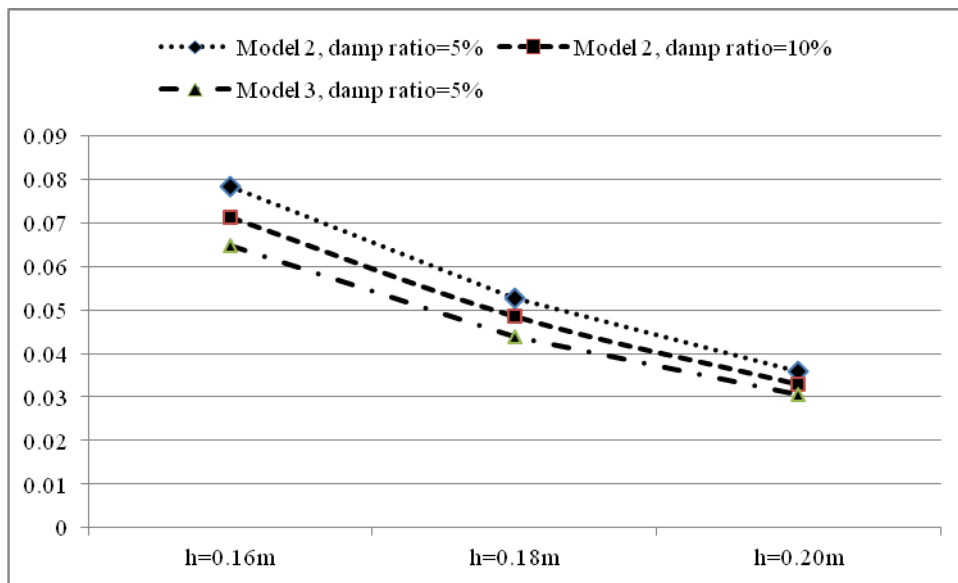


Figure 5. The maximum dynamic deflection at the mid-point of the orthotropic floor plate subjected to a localized blast load for different value of thickness.

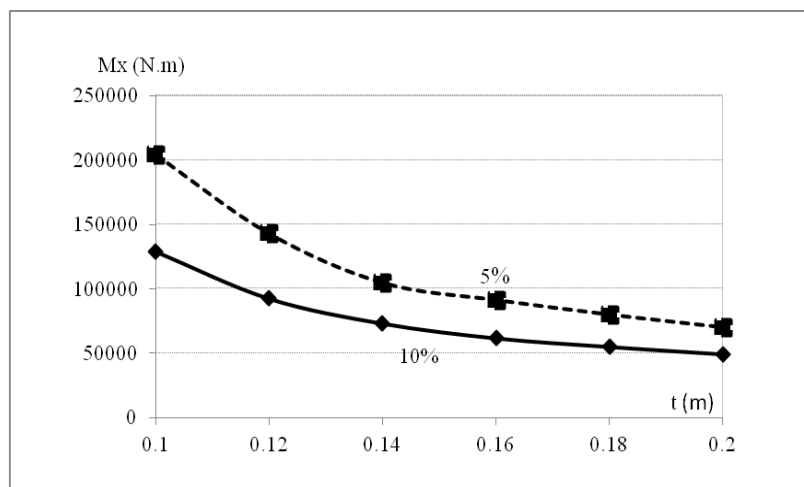


Figure 6. The maximum absolute value of M_x along the x axes as the function of the thickness for model 2 (1 stiffener).

5.3. Effect of Time Duration of the Blast Load

For model 2 with the value of damping ratio of 0%, 5% and 10% increasing the time duration of the triangular load (t_d) by a factor of 2 has resulted in an increase in the mid-point displacement by a factor of 1.87, 1.91 and 1.96 respectively. For model 3 with the value of damping ratio of 0%, 5% and 10% increasing the time duration of the triangular load (t_d) by a factor of 2 has resulted in an increase in the mid-point displacement by a factor of 1.97, 1.94 and 1.99 respectively. Therefore, the time duration of the blast loading plays an important role in determining the level of response of the orthotropic plate.

Table III: The maximum dynamic deflection of a damped orthotropic floor plate subjected to a localized blast load as a function of t_d for model 2 (1 stiffener) and model 3 (2 stiffeners).

Damping ratio (γ)	$t_d=1$ ms w_{max} (m)	$t_d= 2$ ms w_{max} (m)	$t_d= 20$ ms w_{max} (m)
Model 2 (1 stiffener); h=0.16 m			
$\gamma=0$ %	0.00378321	0.00706284	0.0368568
$\gamma=5$ %	0.00278386	0.00533084	0.026842
$\gamma= 10$ %	0.0022917	0.00450229	0.0212333
Model 3 (stiffeners); h=0.16 m			
$\gamma=0$ %	0.00335309	0.0066007	0.0313621
$\gamma=5$ %	0.00242794	0.00472416	0.0229505
$\gamma= 10$ %	0.00197927	0.00394233	0.0133791

5.4. Effect of the Damping Ratio

For model 2 ($t=0.16$ m) with the value of $\gamma=5$ % and $t_d=2$ ms, the absolute dynamic deflection of the system at the mid-point subjected to a localized blast load is 0.00533084 m. By increasing the value of damping ratio with the factor of 2 has resulted in a decrease in the mid-point absolute dynamic deflection by 15.54 %. Increasing the value of damping ratio of the plate has also resulted in a decrease of the distribution of the internal moment in the x direction (M_x) along the x axes for all values of thickness considered in this study, as shown in Figure 5 and Figure 6. Therefore, the damping ratio of the system plays an important role in determining the level of response of the orthotropic plate.

6. CONCLUSIONS

From the dynamic analyses of the orthotropic damped floor plate subjected to a localized blast load the following conclusions can be drawn:

1. The effect of the thickness can be very important, since it affects drastically the overall behaviour of the orthotropic plate.
2. The effect of stiffeners configuration is not as dominant in reducing the overall behaviour of orthotropic floor plate as increasing the thickness of the floor.
3. The time duration of a triangular blast load (t_d) is one of the most important parameters, since it has an influence on other responses, such as moment distribution and the maximum dynamic deflection of the system.

4. The inclusion of damping in calculating the dynamic response of the system will result in a much stiffer responses, especially for large values of t_d resulting in lower mid-point deflection of the orthotropic plate.

While this paper deals mainly with computational results, Kim and Nurick [10] reported on the experimental result on the significance of the thickness of a plate when subjected to localized blast loads. Both approaches provide satisfactory correlation and create better understanding of the localized blast load and the significance of the plate thickness.

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