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DYNAMIC RESPONSE OF RUNWAY PAVEMENT

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Abstract

In this paper the dynamic response of rigid pavements to dynamic moving loads are investigated. To solve this complicated problem, the rigid pavement is modeled as an orthotropic rectangular plate supported by Pasternak foundation. For a simply supported plate, the wave numbers are equal to $m\pi/a$ and $n\pi/b$, where 'a' and 'b' denote the length of the plate in the x and y direction respectively and m and n are positive integers, which determine the number of the mode. The mode shape is presented as a product of eigen functions and is further used in the dynamic response analysis. The dynamic loading function is described as a concentrated moving transverse load of harmonically varying amplitude, which travels with a constant speed. Such a loading may be considered to represent an aircraft wheel loading on a runway pavement upon landing of the aircraft. The general solution for this loading function is derived in integral form. This integral is then solved to determine the forced responses of the plate. It is the purpose of this paper to illustrate and demonstrate the applicability of this theory in practice by presenting numerical results of the analysis of the natural frequencies, dynamic response deflections, bending moments and shear forces of an example rigid runway pavement.

Keywords: Dynamic, rigid pavement, runway

1. Introduction

Several plate elements used in civil engineering, aerospace and marine structures are supported by elastic or viscoelastic media and subjected to transverse dynamic loads. The usual approach in formulating these problems is based on the inclusion of the foundation reaction into the corresponding differential equation of the plate. The foundation is very often a complex medium, but since of interest here is the response of the plate. the problem reduces to finding a relatively simple mathematical expression, which could describe the response of the foundation at the contact area.

A Pasternak foundation model assumes the existence of shear interaction between the spring elements. This may be accomplished by connecting the ends of the springs to a beam or plate consisting of incompressible vertical elements as shown in Fig. (1), which deforms only by transverse shear [1]. Some of the more recent studies dealing with the stiffness analysis of plates resting on an elastics foundation include work by Al-Mahaidi, R. et all [2], who determined the stiffness analysis of plates resting on a Kerr Foundation model. Later, Alisjahbana and Wangsadinata [3] presented a rather general dynamic response of a rigid pavement due to the traffic load rested on a Winkler type foundation, although the effects of the in-plane forces were not discussed.

The purpose of this analysis is to present a general solution based on Fourier techniques for the free and forced response of a runway pavement supported by Pasternak foundation subjected to a landing airplane load p(x,y,t). The runway is modeled as an orthotropic plate supported by a Pasternak foundation.

2. General Analysis

The sides of the rectangular damped orthotropic plate, a and b, are parallel to the x and y axes respectively as shown in Figure 1. The plate is subjected to a general moving transverse dynamic load p(x,y,t) and rests on a Pasternak foundation with a foundation modulus k_1 and the shear foundation modulus G_s. Expressing the plate deflection as w(x,y,t), the general differential equation of the deflected surface is as follows:

$$D_x\frac{\partial^4 w}{\partial x^4}+2B\frac{\partial^4 w}{\partial x^2\partial y^2}+D_y\frac{\partial^4 w}{\partial y^4}+\gamma h\frac{\partial w}{\partial t}+\rho h\frac{\partial^2 w}{\partial t^2}+k_1 w-G_s\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right]=p(x,y,t) \tag{1}$$

where D_x, D_y are flexural rigidities in x and y directions respectively and B is the effective torsional rigidity; γ is the damping ratio and ρ is the mass density.

The solution of the homogeneous orthotropic plate equation can be determined by the method of separation of variables. By substituting separation variables that satisfy the boundary conditions according to:

$$w_{mn}(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}W_{mn}(x,y)T_{mn}(t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}T_{mn}(t) \tag{2}$$

into the homogeneous equation of motion according to Eqn. (1), one obtains:

$$\begin{aligned}&\left[D_x\frac{m^4\pi^4}{a^4}+2B\frac{m^2n^2\pi^4}{a^2b^2}+D_y\frac{n^4\pi^4}{b^4}+k_1+G_s\left[\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right]\right]W_{mn}(x,y)T_{mn}(t)=\\&-\rho h\frac{\partial^2 T_{mn}(t)}{\partial t^2}W_{mn}(x,y)-\gamma h\frac{\partial T_{mn}(t)}{\partial t}W_{mn}(x,y)=\beta_{mn}^4\end{aligned} \tag{3}$$

Since $W_{mn}(x,y)$ depends only on the spatial variables and $T_{mn}(t)$ depends on the temporal variables, each side of Eqn. (3) must be equal to a constant. These separation constant values, or eigenvalues, will be denoted as β^4_{mn} that can be expressed as follows:

$$\beta_{mn}^4=D_x\frac{m^4\pi^4}{a^4}+2B\frac{m^2n^2\pi^4}{a^2b^2}+D_y\frac{n^4\pi^4}{b^4}+k_1+G_s\left[\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right] \tag{4}$$

Furthermore, the natural frequencies of the plate, which are related to the separation constants β^4_{mn} are given by:

$$\omega^2_{mn} = \frac{\beta^4_{mn}}{\rho h} \qquad (5)$$

Thus, the solution of the homogeneous equation can be expressed as

$$w_{mn}(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} W_{mn}(x,y)T_{mn}(t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-\bar{\gamma}\omega_{mn}t}\left[A_{mn}\cos\omega_d t + B_{mn}\sin\omega_d t\right] \qquad (6)$$

where A_{mn} and B_{mn} are coefficient constants that can be determined through the initial conditions, $\omega_d = \omega_{mn}\sqrt{(1-\bar{\gamma}^2)}$ is the damped frequency of the system.

3. Forced Response

Since a fundamental set of solutions of the homogeneous partial differential equation is known and given by the eigenfunctions, it is appropriate to use the method of variation of parameters as a general method of determining a particular solution of the corresponding non-homogeneous partial differential equation of motion.

Using the characteristic function from Eqn.(2), an appropriate solution for the forced response may be written in the form:

$$w_{mn}(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin\left[\frac{m\pi x}{a}\right]\sin\left[\frac{n\pi y}{b}\right]T_{mn}(t) \qquad (7)$$

where $T_{mn}(t)$ is a function of time and must be determined through further analysis.
After substituting Eqn.(7) into the governing non-homogeneous partial differential equation of motion, Eqn.(1) can be put in the following form:

$$\left[D_x\frac{m^4\pi^4}{a^4} + 2B\frac{m^2n^2\pi^4}{a^2b^2} + D_y\frac{n^4\pi^4}{b^4} + k_1 + G_s\left[\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right]\right]W_{mn}(x,y)T_{mn}(t)$$
$$+\bar{\gamma}h\frac{\partial T_{mn}(t)}{\partial t}W_{mn}(x,y) + \rho h\frac{\partial^2 T_{mn}(t)}{\partial t^2}W_{mn}(x,y) = p(x,y,t) \qquad (8)$$

The differential equation for the coefficient functions $T_{mn}(t)$ may be obtained by multiplying both sides of Eqn.(8) in turn by either $\sin\left[\frac{m\pi x}{a}\right]$ or $\sin\left[\frac{n\pi y}{b}\right]$ and integrating over the plate region $0<x<a; 0<y<b$. Thus an ordinary differential equation for $T_{mn}(t)$ is obtained in the following form:

$$\ddot{T}_{mn}(t) + 2\bar{\gamma}\omega_{mn}\dot{T}_{mn}(t) + \omega^2_{mn}T_{mn}(t) = \int_0^a \sin\frac{m\pi x}{a}dx\int_0^b \sin\frac{m\pi x}{a}dy\left[\frac{p(x,y,t)}{\rho h Q_{mn}}\right] \qquad (9)$$

where $\bar{\gamma} = \left[\frac{\gamma}{2\rho\omega_{mn}}\right]$ is a damping factor ratio and Q_{mn} is a normalization factor.

Note that the homogeneous solution of Eqn.(9) is identical with the one previously obtained using the separation of variables solution method. The total solution of Eqn.(9) for $T_{mn}(t)$ is

$$T_{mn}(t) = \hat{T}_{mn}(t) + T^*_{mn}(t) \qquad (10)$$

where $\hat{T}_{mn}(t)$ is the homogeneous solution and $T^*_{mn}(t)$ is the particular solution that can be represented in the form of a Duhamel convolution integral as follows

$$T^{*}_{mn}(t)=\int_0^t\left[\frac{e^{-\bar{\gamma}\omega_{mn}(t-\tau)}}{\rho hQ_{mn}\omega_d}p(x,y,\tau)\int_0^a\left[\sin\frac{m\pi x}{a}dx\right]\int_0^a\left[\sin\frac{n\pi y}{b}dy\right]\sin\omega_d(t-\tau)\right]d\tau \tag{11}$$

The homogeneous solution of the function $\hat{T}_{mn}(t)$ contains the constants that must be determined from the initial conditions, which represents a transient state of vibration motion resulting from the initial conditions. The nature of the steady state forced responses of the plate is contained entirely in the functions $T^{*}_{mn}(t)$ defined by Eqn.(11).

Substituting the expressions for the coefficient functions in Eqn.(11), the general deflection solution for the forced response of an orthotropic rectangular plate to an arbitrary transverse dynamic load p(x,y,t) is given in integral form by

$$w_{mn}(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-\bar{\gamma}\omega_{mn}t}\left[A_{mn}\cos\omega_d t+B_{mn}\sin\omega_d t\right]$$

$$+\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\left[\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\right]\left[\int_0^t\left[\frac{p(x,y,\tau)e^{-\bar{\gamma}\omega_{mn}(t-\tau)}}{\rho hQ_{mn}\omega_d}\int_0^a\left[\sin\frac{m\pi x}{a}\right]dx\int_0^b\left[\sin\frac{n\pi y}{b}\right]dy\right]\sin\omega_d(t-\tau)\right]d\tau \tag{12}$$

The general solution presented above may be integrated to determine the response of the plate for an arbitrary applied transverse dynamic load p(x,y,t).

A concentrated transverse load of harmonically varying amplitude moving in x direction of a plate in a straight line path at a constant y position with a constant speed v, which may be considered to represent an aircraft wheel loading upon landing of the aircraft, can be expressed as follows:

$$p(x,y,t)=P_0(1+\alpha\cos\omega t)\delta[x-vt]\delta[y-y_0] \tag{13}$$

where α is a coefficient which is equal to 0.5.

Substituting the load function given in Eqn.(13) into Eqn.(12), resulting in the general response of the runway pavement subjected to an aircraft wheel load upon landing.

4. Dynamic Response of the Plate

Consider the case of the moving wheel load of an aircraft during landing with constant approaching speed v along the x direction. The load may be expressed as $P_0(1+0.5)\cos\omega t$. At $t = t_0$, in which $t_0 = a/v$, the load leaves the plate. Thus, this problem may be treated in two parts. The first part involves a harmonically oscillating concentrated transverse load moving in x direction at a constant y_0 position. The second part, in which the load is no longer on the plate, involves a free vibration response of the system. The two parts of the problem are related through the boundary conditions. The motion of the plate at $t = t_0$ due to the load at $y = y_0$ becomes the initial condition of the plate at the subsequent instantaneous loading change at $t = t_0$.

Using the above principles, the motion during an interval of time in which the load is no longer on the plate can be computed. Assuming the motion has achieved steady state prior to the load leaving the plate, the motion at $t = t_0$ may be easily computed. This motion at $t = t_0$ determines the initial condition for the second part of the problem. The response of the system can be easily computed by the following equation:

$$w_{mn}(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}W_{mn}(x,y)\left[e^{-\bar{\gamma}\omega_{mn}(t-t_0)}\right]\left[w_{0mn}\cos\omega_d(t-t_0)+\frac{v_{0mn}+\bar{\gamma}\omega_{mn}w_{0mn}}{\omega_d}\sin\omega_d(t-t_0)\right] \tag{14}$$

in which w_{omn} and v_{omn} in Eqn.(14) are the initial deflection and velocity at $t = t_0$.

Bending moments and the vertical shear forces in the plate can be computed in terms of the deflections obtained from Eqn.(14) from the following expressions:

$$M_x = -\left[D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y^2}\right] \quad M_y = -\left[D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x^2}\right] \quad M_{xy} = -2G_{xy} \frac{\partial^2 w}{\partial x \partial y}$$

$$Q_x = -\frac{\partial}{\partial x}\left[D_x \frac{\partial^2 w}{\partial x^2} + B \frac{\partial^2 w}{\partial y^2}\right] \quad Q_y = -\frac{\partial}{\partial y}\left[D_y \frac{\partial^2 w}{\partial y^2} + B \frac{\partial^2 w}{\partial x^2}\right] \tag{15}$$

where D_x, D_y, D_{xy} and G_{xy} represent the flexural rigidities and the torsional rigidity for an orthotropic plate, $B = D_{xy} + 2G$ and G is the elastic shear modulus of the plate.

4. Numerical Example

Using the procedure described above, a runway pavement subjected to the moving dynamic wheel load of an aircraft during landing will be analyzed. The effect of changing the load's frequency ω and the damping ratio γ will be considered. The transverse dynamic load is $P_0 = 2\text{x}10^5$ N, traveling with a constant approaching speed v = 260 km/hr. along the x direction representing the wheel loading of a DC 10 aircraft 30/40 series during landing. The following numerical results have been calculated for the following case: a = 7.5m, b = 15m, $\rho = 2.4\text{x}10^3$ kg/m^3, h = 0.5m, $E_x = 30\text{x}10^9$N/m^2, $E_y = 20\text{x}10^9$N/m^2, $\nu_x = 0.2$, $\nu_y = 0.1$, G = 10^{10}N/m^2, $k_1 = 7.5\text{x}10^7$ N/m^2/m, $G_s = 2.5\text{x}10^8$N/m^2, $y_o = 7.5$ m.

Tabel 1. Natural frequencies of the runway plate for the first 5 modes (m=1,2,...,5 and n=1,2,...,5).

n	m=1 ω_{mn} (rad/sec)	n	m=2 ω_{mn} (rad/sec)	n	m=3 ω_{mn} (rad/sec)	n	m=4 ω_{mn} (rad/sec)	n	m=5 ω_{mn} (rad/sec)
1	346.115	1	601.178	1	1044.68	1	1673.45	1	2485.13
2	401.956	2	656.445	2	1098.67	2	1726.52	2	2537.62
3	494.798	3	748.598	3	1188.94	3	1815.28	3	2625.37
4	624.513	4	877.668	4	1315.8	4	1940.1	4	2748.71
5	791.06	5	1043.68	5	1479.51	5	2101.37	5	2908.05

Table 1 shows the natural frequencies of the system for the first 5 modes (m=1,2,...,5 and n=1,2,...,5). It can be seen from the table that the natural frequency increases as the mode number increases. Figure 2 shows the dynamic response spectra as a function of the load's frequency and damping ratio. It can be seen that the dynamic deflection will be maximum when the load's frequency approaches the value of the first natural frequency of the runway plate.

Figure 3 shows the various responses of the runway plate to the moving transverse wheel loading of the aircraft. By comparing the case at near resonance condition and that away from resonance condition, one can recognize the significance of avoiding the resonance condition, since at resonance the various responses are apparently relatively very high.

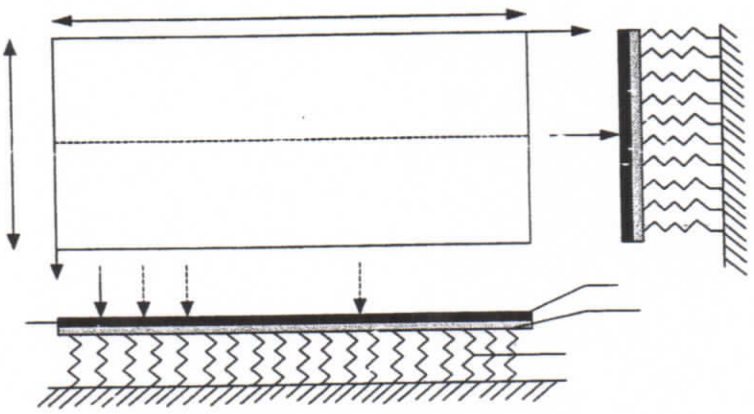


Figure 1. A rectangular orthotropic plate resting on a Pasternak foundation subjected to a general moving transverse dynamic load.

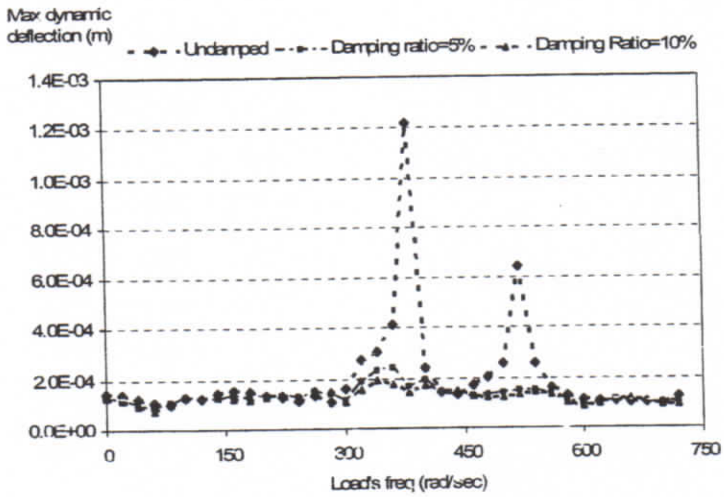
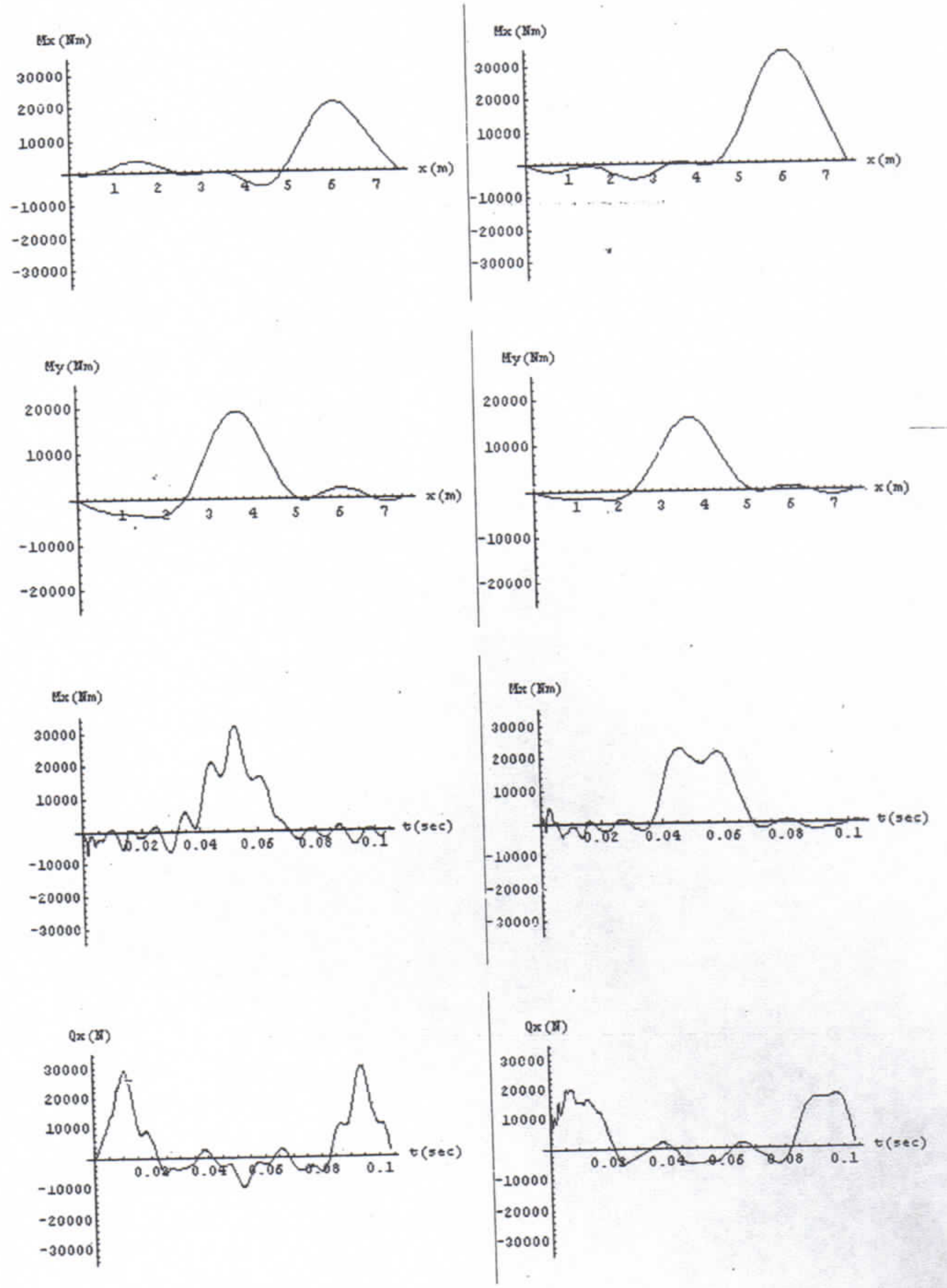


Figure 2. Maximum dynamic deflection response spectra as a function of the load's frequency and damping ratio.


Mx (Nm)
30000
20000
10000
-10000
-20000
-30000
x (m)
1 2 3 4 5 6 7
My (Nm)
20000
10000
-10000
-20000
x (m)
1 2 3 4 5 6 7
Mx (Nm)
t (sec)
0.02 0.04 0.06 0.08 0.1
Qx (N)
t (sec)
0.02 0.04 0.06 0.08 0.1

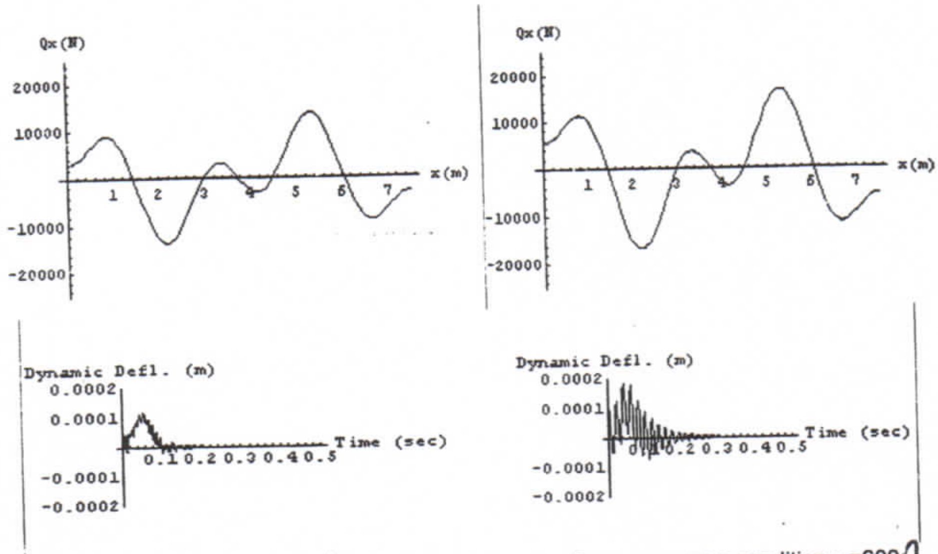


Figure 3. Various dynamic responses of the plate away from resonance condition ω=600 rad/sec (left) and at near resonance condition ω=300 rad/sec (right).

5. Conclusions

In conclusion the following can be stated:

1. The theory of the orthotropic rectangular plate supported by a Pasternak foundation subjected to a moving transverse dynamic load based on Fourier techniques can reasonably be applied for the analysis of rigid pavements, such as runway pavements, subjected to aircraft wheel loading during landing of the aircraft.
2. This dynamic response analysis gives a better understanding of plate behavior under the effect of the moving transverse dynamic loads, so that it becomes an additional design tool beside the conventional static design approach.
3. This dynamic response design approach would give more freedom in the selection of pavement and foundation material properties, since it is the combined material effect, rather than the individual ones, that determines the overall performance of a rigid pavement that is shown from the result of the dynamic response analysis.
4. For certain aircraft loadings, impact characteristics upon landing and approaching speeds, it is possible to construct response spectra design charts, which is the subject of further study of the authors.

6. References

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