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THE SIGNIFICANCE OF THE THICKNESS AND THE STRUCTURAL DAMPING OF AN ORTHOTROPIC PLATE WHEN SUBJECTED TO LOCALIZED BLAST LOADS

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Abstract. *An investigation has been conducted to examine the dynamic behavior of orthotropic plates with different values of plate thickness and structural damping ratio to localized blast loads. The orthotropic plates are of 5.5 m and 4.5 m dimensions, with partially fixed boundary conditions along all edges. The aim of this work is to determine the dynamic response of orthotropic plates to localized blast loads, and to assess the significance of plate thickness and structural damping ratio on deflection of the plate. The numerical solutions of the natural frequencies are solved from two transcendental equations while the eigen functions of the system were solved by using the Modified Bolotin Method (MBM). Localized blast loading is further integrated by using the Duhamel integration to find the total dynamic response of the system. Special emphasis is focused on the maximum absolute dynamic deflections of the plate under localized blast load. The results obtained give an insight into the effect of the significance of the plate thickness and structural damping on the response of the orthotropic plate under localized blast loads and indicate that plate thickness and structural damping can affect their overall dynamic behavior.*

1 INTRODUCTION

Due to different accidental and intentional events, the behaviour of an orthotropic floor plate as a part of structural components subjected to blast loading has been the subject of considerable research effort in recent years.

To provide adequate protection against blast loading, the design and construction of hospitals, schools, office buildings have received renewed attention of structural engineers.

Louca and Harding[1], Kadit et al[2] had presented analyses for plates subjected to blast loading, which included the effect of the stiffeners configurations to the plate's response. Most of the researches were on simply supported orthotropic plates subjected to blast loading modelled as a triangular function, an exponential function and a stepped triangular function. Problems dealing with the response of orthotropic floor plates to blast loading with non simply supported condition were more complicated to be numerically solved. What is most relevant to the present work is the use of the Modified Bolotin Method (MBM) for solving the free vibration modes of rectangular plates (Pevzner et al[3]).

Alisjahbana and Wangsadinata[4] had extensively studied the dynamic response of orthotropic

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floor plate with a general restraint condition along its support subjected to blast load with different stiffeners configuration.

In the present research work the problem of an orthotropic stiffened floor plate under a localized blast load is further studied, whereby the plate is partially fixed along its support. The dynamic response of the orthotropic stiffened floor plate is computed for the different thickness value and different values of damping ratio. The vibration modes are solved using the Modified Bolotin Method and the mode shapes are expressed as a product of eigen functions. The mode numbers in the *x* and in the *y* directions are solved from the transcendental equations which satisfy the boundary conditions.

The geometry and material properties of the plate are assumed to be linear elastic and orthotropic and of finite dimensions. Finally, results on dynamic responses such as midpoint deflection, bending moments and shear forces of the orthotropic plate are presented incorporating the effects of the plate thickness.

2 GOVERNING EQUATIONS

Using the classical theory of thin plates, the equation of equilibrium of an elastic orthotropic stiffened plate is as follows:

$$D_x \frac{\partial^4 w(x,y,t)}{\partial x^4} + 2B \frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w(x,y,t)}{\partial y^4} + \gamma h \frac{\partial w(x,y,t)}{\partial t} + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = p(x,y,t) \tag{1}$$

where D_x and D_y are the flexural rigidity in x and y direction respectively, B is the torsional rigidity, γ is the damping ratio, ρ is the mass density of the plate and p(x,y,t) is the blast load. The plate is stiffened by stiffeners parallel to x axes and the origin of the Cartesian coordinates (x,y) is set at the lower left corner of the plate, w(x,y,t) is the transverse deflection of the mid surface. The two considered types of support conditions for each plate edges are as follow:

Along x=0 and x=a

$$-D_x \left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \nu_y \frac{\partial^2 w(x,y,t)}{\partial y^2} \right) = k_1 \frac{\partial w(x,y,t)}{\partial x}; \quad w(x,y,t) = 0 \tag{2}$$

Along y=0 and y=b

$$-D_y \left(\frac{\partial^2 w(x,y,t)}{\partial y^2} + \nu_x \frac{\partial^2 w(x,y,t)}{\partial x^2} \right) = k_2 \frac{\partial w(x,y,t)}{\partial y}; \quad w(x,y,t) = 0 \tag{3}$$

where k_1 is an elastic rotational restraint constant along x=0 and x=a and k_2 is an elastic rotational restraint along y=0 and y=b. A model of an orthotropic stiffened plate with rotational restraints along its edges subjected to a blast loading can then be established.

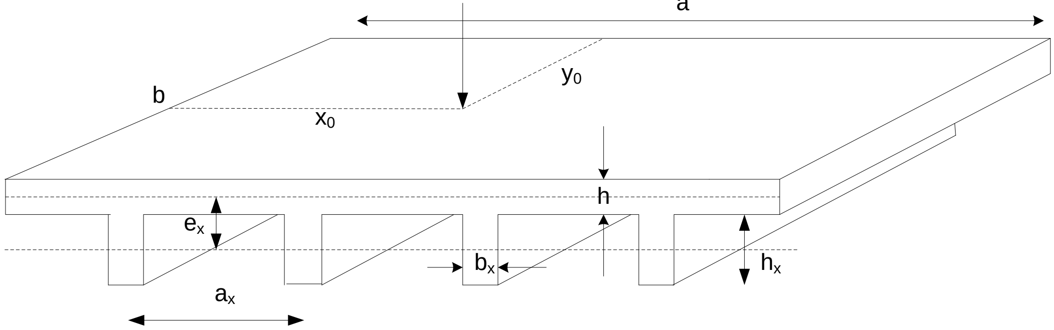


Figure 1. Rectangular orthotropic stiffened plate subjected to dynamic load p(x,y,t).

A localized blast load modeled as a triangular pulse function can be expressed by the following expression:

$$p(x,y,t) = P(t)\delta[x - x(t)]\delta[y - y(t)] = P(t)\delta[x - x_0]\delta[y - y_0] \quad (4)$$

$$P(t) = P_0\left(1 - \frac{t}{t_d}\right) \text{ for } 0 \le t \le t_d \quad (5)$$

$$P(t) = 0 \text{ for } t > t_d \quad (6)$$

where P_0 =the maximum amplitude of the load; t_d= time duration; x_0= position of the localized blast load in x direction; y_0= position of the localized load in y direction.

3 NATURAL FREQUENCIES

The free vibration of the orthotropic stiffened floor plate with semi rigid condition along its support is studied first using the Levy's solution. The free vibration of the system is set as:

$$w(x,y,t) = W(x,y)T(t)\sin\omega t \quad (7)$$

where *W(x,y)* is a function of the position coordinates only, and ω is the circular frequency.

The undamped free vibration equation of motion of the system can be expressed as:

$$D_x \frac{\partial^4 W(x,y)}{\partial x^4} + 2B\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W(x,y)}{\partial y^4} + \rho h \omega^2 W(x,y) = 0 \quad (8)$$

The next step is to find the solution of Eq. (8) with the boundary conditions according to Eq. (2) and Eq. (3), to obtain the eigen frequencies and the mode shapes of the orthotropic floor plate. By postulating the following eigen frequency, which analogous to the case of a plate simply supported at all edges (Pevzner et al, 2000), natural frequencies of the system can be expressed as:

$$\omega_{mn} = \sqrt{\left(\frac{\pi^4}{\rho h}\right)\left[D_x\left(\frac{p}{a}\right)^4 + 2B\left(\frac{pq}{ab}\right)^2 + D_y\left(\frac{q}{b}\right)^4\right]} \quad (9)$$

where *p* and *q* are real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problem.

4 DYNAMIC RESPONSE

The dynamic response of the stiffened orthotropic floor plate can be found by using the method of variation of parameters as a general method of determining a particular solution of the corresponding non-homogeneous partial differential equation, which can be written in the following form:

$$w_{mn}(x,y,t) = \sum_{m=1}^{m}\sum_{n=1}^{n} X_m(x)Y_n(y)T_{mn}(t) \quad (10)$$

where $X_m(x)$, $Y_n(y)$ are eigen functions, $T_{mn}(t)$ is a function of time, which must be determined through further analyses.

The differential equation for the function $T_{mn}(t)$ can be expressed as:

$$\ddot{T}_{mn}(t) + 2\gamma\omega_{mn}\dot{T}_{mn}(t) + \omega_{mn}{}^2 T_{mn}(t) = \int_0^a X_m(x)dx\int_0^b Y_n(y)dy\,\frac{p(x,y,t)}{\rho h Q_{mn}} \quad (11)$$

$Q_{mn}(x)$ is a normalization factor that can be expressed as:

$$Q_{mn} = \int_0^a (X_m(x))^2 dx \int_0^b (Y_n(y))^2 dy \quad (12)$$

The particular solution of the temporal function $T_{mn}(t)$ can be represented in a form of the Duhamel convolution integral as follows:

$$T_{mn}^*(t) = \int_0^t \left[\frac{p(x,y,\tau)}{\rho h Q_{mn}} \int_0^a X_m(x)dx \int_0^b Y_n(y)dy \, \frac{p(x,y,t)}{\rho h Q_{mn}} \frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}} \sin\omega_{mn}(t-\tau) \right] d\tau \quad (13)$$

The general solution for the forced response deflection of the orthotropic stiffened floor plate to a localized blast load p(x,y,t) is given in integral form as follows:

$$w(x,y,t) = \sum_{m=1}^{m}\sum_{n=1}^{n} X_m(x)Y_n(y) \int_0^t \left[\left[\frac{P(\tau)\delta[x-x_0]\delta[y-y_0]}{\rho h Q_{mn}} \int_0^a X_m(x)dx \int_0^b Y_n(y)dy \right] \frac{p(x,y,t)}{\rho h Q_{mn}} \frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}} \sin\omega_{mn}(t-\tau) \right] d\tau \quad (14)$$

Once the response deflections of the orthotropic stiffened floor plate have been obtained, the internal forces of the floor (moment and shear forces) can be computed, using derivatives of those deflections.

5 NUMERICAL RESULTS

A reinforced concrete rectangular damped plate stiffened by rectangular stiffeners parallel to the x axes is considered. The material is assumed to be orthotropic and linearly elastic. The data for the orthotropic floor plate and blast load are: a=5.5 m, b=4.75 m, E_c=2.57E^9 N/m^2, ρ=2400 kg/m^3, P_0=1.3E^6 N/m, t_d=1 ms, x_0=1/3a, y_0=1/3b. The absolute maximum dynamic deflection of the plate at mid plate due to a localized blast load will be calculated for γ=5% and γ=10% by using 5 modes in the x direction (m=1,2,...,5) and 5 modes in the y direction (n=1,2,...,5). Two transcendental equations will be used to obtain the values of p and q and the natural frequencies of the orthotropic plate for six different values of thickness for model 1 (1 stiffener) and model 2 (2 stiffeners) will be obtained. The natural frequencies of the plate are shown in Table 1.

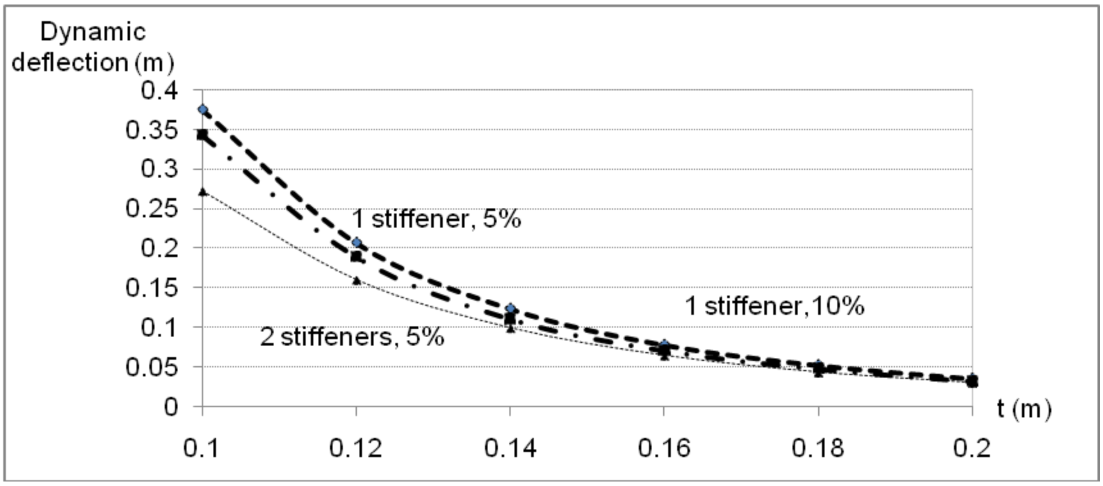


Figure 2.The maximum dynamic deflection at the mid-point of the orthotropic floor plate subjected to a localized blast load for different value of thickness.

m	n	t=0.10m ω_{mn} (rad/s)	t=0.12m ω_{mn} (rad/s)	t=0.14m ω_{mn} (rad/s)	t=0.16m ω_{mn} (rad/s)	t=0.18m ω_{mn} (rad/s)	t=0. 20m ω_{mn} (rad/s)
1	1	148.251	169.867	192.269	215.374	235.545	263.172
	2	348.911	404.346	462.745	522.938	580.903	646.777
	3	636.952	747.495	861.909	978.662	1093.47	1217.27
	4	1016.69	1201.7	1391.31	1583.72	1773.87	1975.9
	5	1488.97	1767.93	2051.76	2338.93	2622.91	2923.87
2	1	319.177	338.605	367.304	398.86	428.117	467.795
	2	520.01	572.575	633.545	699.947	762.674	842.914
	3	818.376	920.951	1035.39	1157.14	1275.48	1414.38
	4	1206.25	1380.06	1568.17	1764.96	1958.54	2175.76
	5	1684.86	1950.18	2231.56	2522.76	2810.4	3126.48
3	1	610.632	646.899	691.595	742.21	793.79	855.227
	2	813.649	871.813	944.083	1025.84	1105.92	1207.6
	3	1119.21	1228.99	1341.11	1475.27	1606.84	1768.3
	4	1516.87	1682.41	1873.53	2080.74	2285.28	2525.88
	5	2004.6	2256.46	2538.24	2838.57	3136.06	3475.96
4	1	1031.16	1086.78	1156.67	1236.47	1320.9	1415.69
	2	1230.45	1303.72	1397.51	1505.32	1615.31	1748.27
	3	1539.61	1648.65	1786.19	1942.72	2099.83	2292.32
	4	1945.35	2111.82	2314.41	2540.78	2767.09	3039.25
	5	2443.59	2688.66	2977.65	3294.66	3611.26	3983.31
5	1	1573.56	1655.4	1758.84	1877.23	2004.34	2143.49
	2	1769.71	1866.4	1991.3	2135.5	2286.55	2461.82
	3	2080.24	2207.34	2371.69	2561.26	2756.06	2989.61
	4	2492.76	2669.62	2893.88	3149.67	3409.91	3722.64
	5	2999.89	3247.94	3553.47	3896.59	4243.37	4656.46

Table 1: The fundamental frequencies of the orthotropic floor plate (model 1) for the first 5 modes in the *x* direction (*m*=1,2,...,5) and for the first 5 modes in the *y* direction (*n*=1,2,...,5).

Table 1 shows the natural frequencies of the orthotropic floor plate for different floor thickness. It is shown that by increasing the plate thickness, the natural frequencies of the system increased.

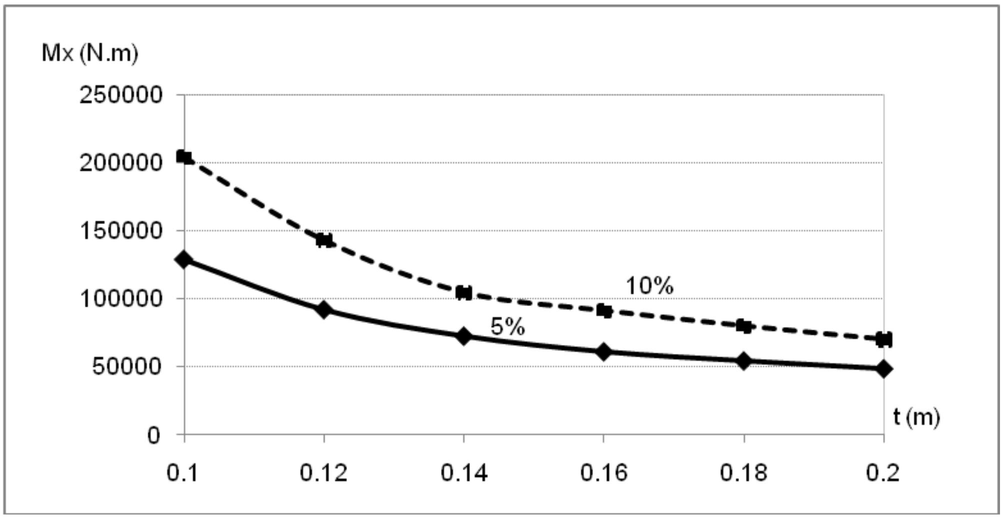


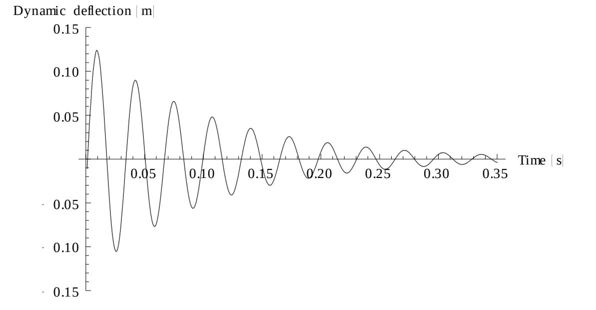
Figure 3. Maximum absolute value of M_x along the x axes as the function of the thickness.

5.2. Effect of damping ratio

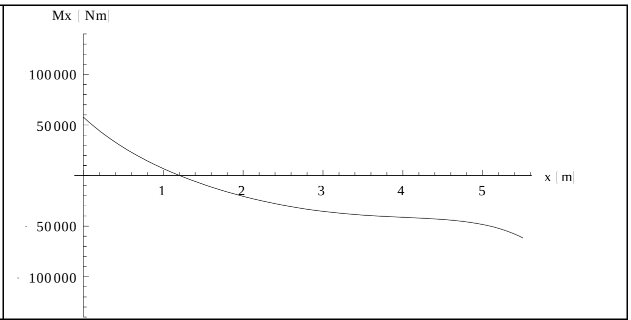
For model 1 (t=0.16 m) with the value of γ=5 % the absolute dynamic deflection of the system at the mid-point subjected to a localized blast load is 0.374755 m. By increasing the value of damping ratio with the factor 2 has resulted in a decrease in the mid-point absolute dynamic deflection by 8.9%. Increasing the value of damping ratio of the floor has also resulted in a decrease of the distribution of the internal moment in the x direction (M_x) along the x axes for all values of thickness considered in this study, as shown in Figure 3 and Figure 4. Therefore, the damping ratio of the floor system plays an important role in determining the level of response of the orthotropic floor.

5.3. Effect of stiffeners configuration

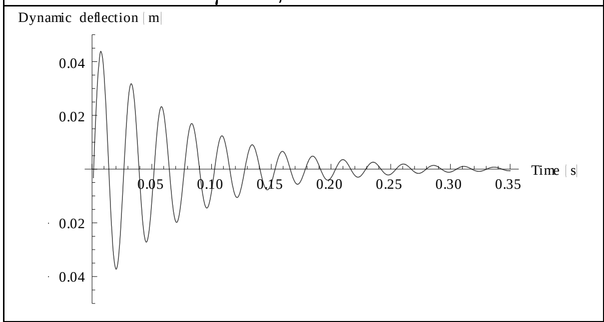
The absolute maximum dynamic deflection of the floor plate has been computed for 2 different stiffeners configuration as shown at Table 2. The existence of the stiffeners in the system decreases the mid-point displacement significantly; the mid-point displacement for model 1 (1 stiffener) for t=0.16 m, γ=5% is 0.0783806 m, while the mid-point displacement for model 2 (2 stiffeners) for t=0.16 m, γ=5% is 0.0647376 m. Therefore the existence of the stiffeners in the system reduces the mid-point deflection of the system by 17.4%.



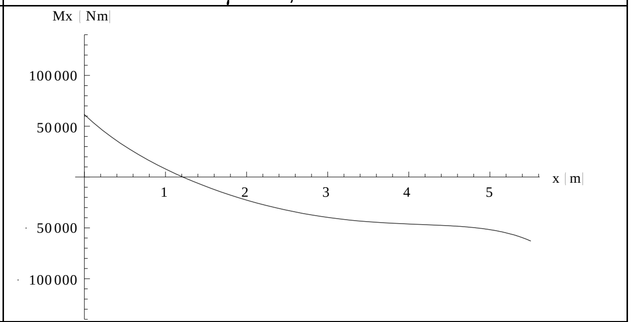
γ=5%, t=0.14 m



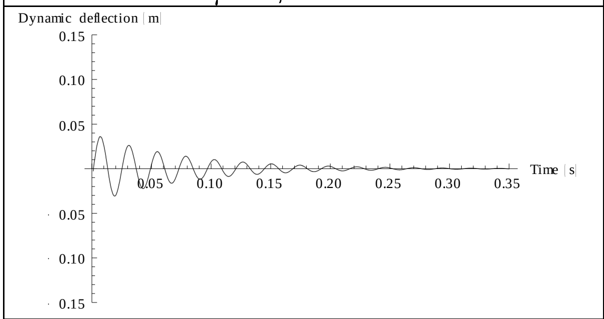
γ=5%, t=0.14 m



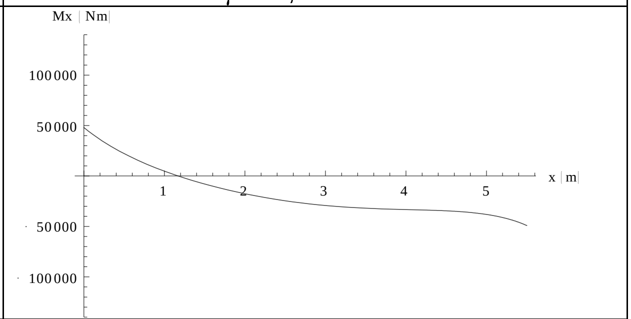
γ=5%, t=0.16 m



γ=5%, t=0.16 m



γ=5%, t=0.18 m

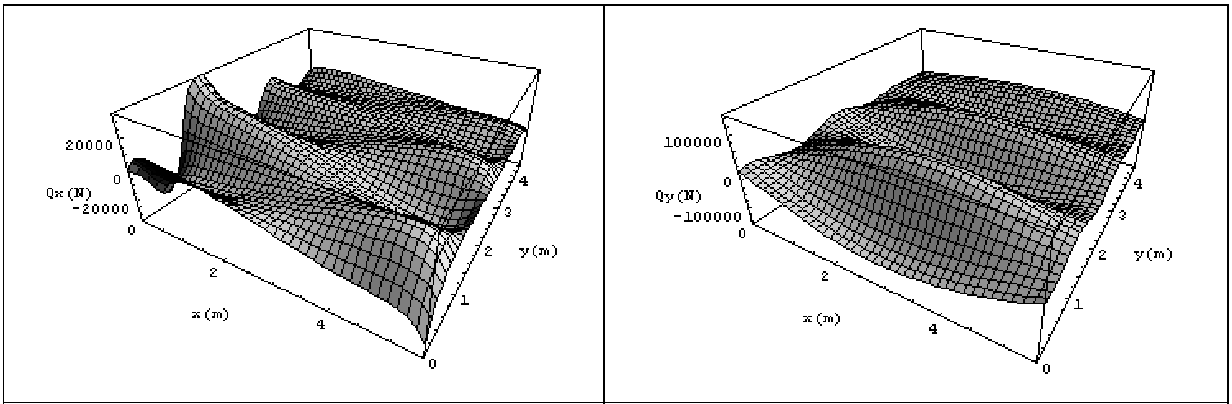


γ=5%, t=0.18 m

Figure 4. Dynamic deflection time history at the mid-point and moment-x (M_x) distribution along x axes for model 1 subjected to a localized blast load.

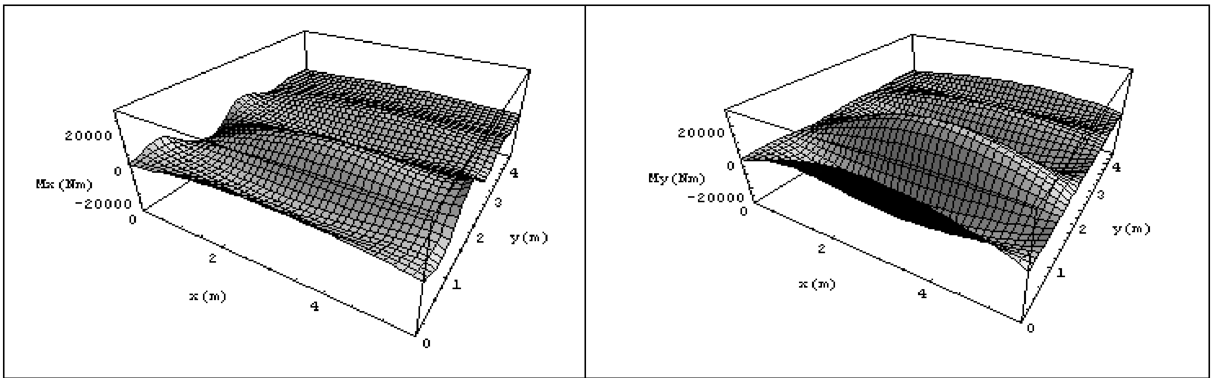
t (m)	w $_{max}$ (m) γ=5%, model 1	w $_{max}$ (m) γ=10%, model 1	w $_{max}$ (m) γ=5%, model 2
0.16	0.0783806	0.0713922	0.0647376
0.18	0.0527863	0.0485378	0.0437584
0.2	0.0358481	0.0328331	0.0305427

Table 2: The maximum dynamic deflection of a damped orthotropic floor plate subjected to a localized blast load as a function of thickness for model 1 (1 stiffener) and model 2 (2 stiffeners).



Q_x distribution	Q_y distribution

Figure 5. Q_x and Q_y distribution along x axes and y axes for model 1 subjected to a localized blast load, γ=10%, t_d=2 ms.



M_x distribution	M_y distribution

Figure 6. M_x and M_y distribution along x axes and y axes for model 1 subjected to a localized blast load, γ=10%, t_d=2 ms.

6 CONCLUSIONS

From the dynamic analyses of the orthotropic damped floor plate subjected to a localized blast load the following conclusions can be drawn:

1. The effect of the thickness can be very important, since it affects drastically the overall behaviour of the orthotropic floor plate.
2. The inclusion of damping in calculating the dynamic response of the system will result in much stiffer responses, especially for model 2 with 2 stiffeners.
3. The effect of stiffeners configuration is not as dominant in reducing the overall behaviour of orthotropic floor plate as increasing the thickness of the floor.

While this paper deals mainly with computational results, Kim and Nurick[5] reported on the experimental result on the significance of the thickness of a plate when subjected to localized blast loads. Both approaches provide satisfactory correlation and create better understanding of the localized blast load and the significance of the plate thickness.

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