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)ynamic response of damped orthotropic plate on Pasternak foundation) dynamic moving loads

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BSTRACT: The study of the dynamic response of a damped orthotropic rectangular plate resting on a asternak foundation subjected to a dynamic moving load is important, as some of the results may contribute the understanding of the dynamic behavior of rigid pavements. In this paper, a procedure incorporating the lodified Bolotin Method is developed to find the natural frequencies and the eigenfunctions of a damped ctangular orthotropic plate supported by an elastic foundation. The effect of a dynamic moving load is further udied, whereby the load is suddenly moving to a new position and continues moving to the edge of the plate ith a constant velocity. The dynamic response of the plate is obtained on the basis of orthogonality properties f eigenfunctions. The dynamic response of the plate is expressed in integral form, so that it can be readily itegrated using the Duhamel's integration method to obtain the various dynamic responses of the plate.

INTRODUCTION

he investigation of the dynamic behavior of an rthotropic plate resting on a Pasternak foundation nder moving loads has been a topic of interest for well ver a century. Some of the results has contributed to e understanding of the dynamic behavior of rigid avements. In the analysis of rigid pavements, the tructure is usually modeled as an orthotropic plate esting on an elastic foundation.

Static and free vibration analyses of a rectangular late on an elastic foundation have received considrable attention in the literature, for example by Saha 1997) and Matsunaga (2000). Dynamic response of rectangular plate on an elastics foundation such as a asternak foundation has attracted much less attention comparison. Gbadeyan & Oni (1992) gave a closed orm solution by using double Fourier sine integral ansformation to analyze a simply supported rectanular plate resting on a Pasternak foundation subjected an arbitrary number of moving concentrated loads. lates with arbitrary boundary conditions are diffiult to solve and numerical procedures have to be mployed.

Recently, Alisjahbana & Wangsadinata (2005) preented the case of a rectangular damped orthotropic late resting on a Pasternak foundation clamped at all dges subjected to a dynamic load. For this clamped late, the wave numbers were presented in the form π/a and $q\pi/b$, where a and b were the side dimenions of the plate, while p and q were real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method (Pevzner et al. 2000).

In the present research work the problem of an orthotropic rectangular plate on a Pasternak foundation under a dynamic moving load is further studied, whereby the plate is under a very general restraint condition along its supports. The vibration modes are solved using the Modified Bolotin Method. As the mode shapes are expressed as a product of eigenfunctions, the dynamic solution of the plate is obtained on the basis of orthogonality properties of eigenfunctions. The dynamic response of the plate is expressed in integral form, so that it can be readily integrated to obtain the various dynamic responses of the plate.

2 GOVERNING EQUATION

Using the classical theory of thin plates, the following equations of equilibrium are obtained for an elastic plate resting on a Pasternak foundation:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2B\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \gamma h \frac{\partial w}{\partial t} + \rho h \frac{\partial^2 w}{\partial t^2} + k_f w - G_s\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] = p(x,y,t) \quad (1)$$

where D_x, D_y = plate stiffnesses in x and y direction; B = plate torsional stiffness; γ = damping ratio; h = plate thickness; ρ = plate mass density; k_f = stiffness of the foundation; G_s = shear modulus of the foundation; t = time; x,y = rectangular Cartesian coordinates in the plane of the plate; $p(x,y,t)$ = dynamic load acting on the plate. The harmonic concentrated load which suddenly moves to a new position and continues to move to the edge of the plate with a constant velocity can be expressed as

$$p(x,y,t) = P_0(1+\alpha\cos\omega t)\delta[x-x(t)]\delta[y-y(t)] \tag{2}$$

where P_0 = amplitude of load; α = load coefficient = 0.5; ω = angular frequency of load; $x(t)$ = position of load in x direction and $y(t)$ = position of load in y direction.

Consider the following types of support conditions for the plate edge:

along $x = 0$

$$-D_x\left(\frac{\partial^2 w}{\partial x^2}+v_y\frac{\partial^2 w}{\partial y^2}\right)=k_1\frac{\partial w}{\partial x} \quad ;w=0 \tag{3}$$

along $x = a$

$$-D_x\left(\frac{\partial^2 w}{\partial x^2}+v_y\frac{\partial^2 w}{\partial y^2}\right)=k_2\frac{\partial w}{\partial x} \quad ;w=0 \tag{4}$$

along $y = 0$ and $y = b$

$$-D_y\left(\frac{\partial^2 w}{\partial x^2}+v_x\frac{\partial^2 w}{\partial x^2}\right)=0; w=0 \tag{5}$$

where ν_x is the Poisson's ratio in x direction, ν_y is the Poisson's ratio in y direction, k_1 is an elastic rotational restraint constant along $x = 0$; k_2 is an elastic rotational restraint along $x = a$. A model of an orthotropic damped plate with unsymmetrical restrained along its edges resting on an elastic foundation subjected to moving loads can then be established.

3 GENERAL ANALYSES

In order to solve the problem described above, it is assumed that the principal elastic axes of the material are parallel to the edges and the free vibration solution of the problem is set as:

$$w(x,y,t) = W(x,y)\sin\omega_{mn}t \tag{6}$$

where $W(x,y)$ is a function of the position coordinates only and ω_{mn} is the circular frequency of the system. Substituting Equation 6 into the homogeneous part of Equation 1, gives the natural frequencies of the system that can be expressed by:

$$\omega^2_{mn} = \left(\frac{\pi^4}{\rho h}\right)\left[D_x\left(\frac{p}{a}\right)^4+2B\left(\frac{pq}{ab}\right)^2+D_y\left(\frac{q}{b}\right)^4\right] + \frac{k_f}{\rho h}+\frac{G_s}{\rho h}\left[\left(\frac{p}{a}\right)^2+\left(\frac{q}{b}\right)^2\right] \tag{7}$$

where a and b are side dimensions of the plate, while p and q are real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method (Pevzner et al., 2000).

4 DYNAMIC RESPONSE OF THE PLATE

The dynamic response of the plate can be found by using the method of variation of parameters as a general method of determining a particular solution of the corresponding non-homogeneous partial differential equation, which can be written in the following form:

$$w_{mn}(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} X_m(x)Y_n(y)T_{mn}(t) \tag{8}$$

where $X_m(x)$, $Y_n(y)$ are eigenfunctions, $T_{mn}(t)$ is a function of time which must be determined through further analysis.

The differential equation for the coefficient functions $T_{mn}(t)$ can be expressed as:

$$\ddot{T}_{mn}(t)+2\gamma\omega_{mn}\dot{T}_{mn}(t)+\omega^2_{mn}T_{mn}(t) = \int_0^a X_m(x)dx\int_0^b Y_n(y)dy\frac{p(x,y,t)}{\rho h Q_{mn}} \tag{9}$$

where Q_{mn} is a normalization factor.

The particular solution of the temporal function $T_{mn}(t)$ can be represented in a form of the Duhamel convolution integral as follows:

$$T^*_{mn}(t) = \int_0^t\left[\frac{p(x,y,\tau)}{\rho h Q_{mn}}\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\right] \left[\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}}\sin\left(\omega_{mn}\sqrt{(1-\gamma^2)}(t-\tau)\right)\right]d\tau \tag{10}$$

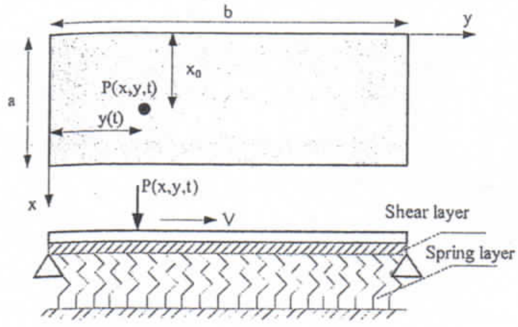

Figure 1. Rectangular orthotropic plate on a Pasternak foundation subjected to moving load.

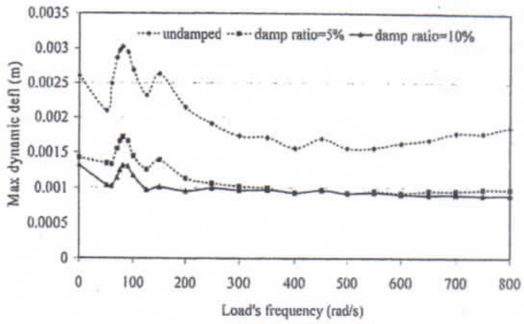

Figure 2. Response spectra as a function of load's frequency for various values of damping ratios.

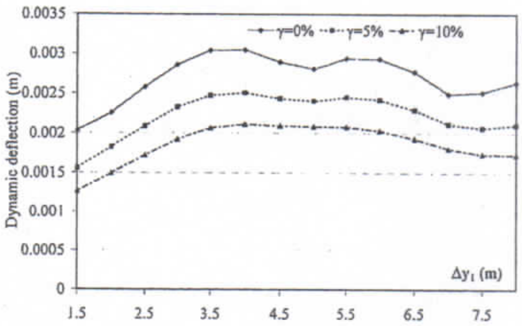

Figure 3. Response spectra as a function of the magnitude of the sudden y position change (Δy_1) for various values of damping ratios.

The general solution for the forced response deflection of the plate to an arbitrary dynamic moving load p(x,y,t) is given in integral form as follows:

$$w(x,y,t)=\sum_{m=1}^{m=\infty}X_m(x)\sum_{n=1}^{n=\infty}Y_n(y)$$

$$\int_0^t\left[\frac{p(x,y,\tau)}{\rho hQ_{mn}}\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\right] \tag{11}$$

$$\left[\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\gamma^2)}}\sin\left(\omega_{mn}\sqrt{(1-\gamma^2)}\,(t-\tau)\right)\right]d\tau$$

Bending moments and vertical shear forces in the plate can be computed in terms of the deflection and its derivatives obtained from Equation 11.

5 RESULTS

An orthotropic rectangular plate resting on an elastic Pasternak foundation subjected to a dynamic moving load is considered. The data for the plate and load amplitude for the numerical examples treated in this section are as follows: $a=7.5$ m, $b=15$ m (Fig. 1), $h=0.5$ m, $E_x=3.0\times10^{10}$ N/m², $E_y=2.0\times10^{10}$ N/m², $G_p=1.09\times10^{10}$ N/m², $\nu_x=0.15$, $\nu_y=0.1$, and $P_0=6.6\times10^5$ N. The boundary conditions are: partially fixed along the longitudinal edges (x = 0, 7.5 m) and simply supported along the shorter edges (y = 0, 15 m). In the following discussion, x_0 and y_0 refer to the moving load position and x and y to the coordinates parallel to the shorter and longer dimensions of the plate, respectively.

In this numerical example, the Pasternak elastic foundation stiffness are given as $k_f=2.72\times10^7$ N/m³ and $G_s=5.4\times10^6$ N/m³. The load moves along the centerline ($x_0=3.75$ m), parallel to the y axis with constant amplitude and constant velocity v = 77 m/s. At $t=t_0=0.01$ s, the load suddenly moves to a new position at $y_1=2.0$ m and continues to move to the other side of the plate.

Due to the mechanical systems of the vehicle, the moving loads exerted by the vehicle might not have a constant amplitude, and a moving harmonic load model is generally used in practical analysis (Kim & Roesset 1998). In this paper a single moving harmonic concentrated load $P_0(1+\alpha\cos\omega t)$, traveling along the middle line ($x_0=3.75$ m) is considered. Figure 2 shows the dynamic deflection under moving harmonic load when the moving velocity is 77 m/s and foundation stiffness is given as $k_f=2.72\times10^7$ N/m³ for various damping ratio. It can be seen that the damping ratios play an important factor to reduce the dynamic deflection of the system.

Figure 3 illustrates the effect of the magnitude of the sudden y position change on the maximum deflection under the moving load for various damping ratios. As expected, the maximum dynamic deflection increases as the change in y position increases for all values of damping ratio.

The bending moments and shear forces change with the increase of the damping ratio for the value of $\omega=600$ rad/s which is far away from the first natural frequency of the system (Figs 4–9).

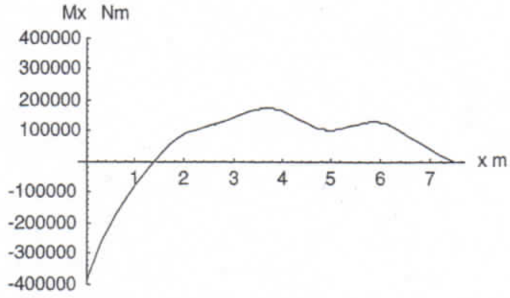

Figure 4. M_x distribution along the x-axis for $\gamma = 0\%$, $\omega = 600$ rad/s computed at $t = 0.0115$ s.

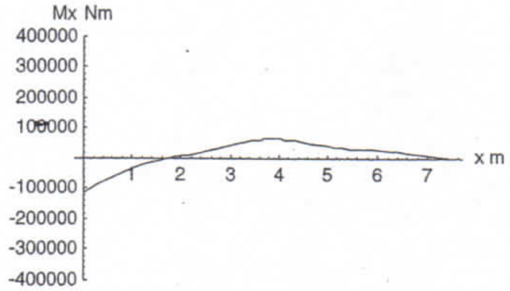

Figure 5. M_x distribution along the x-axis for $\gamma = 10\%$, $\omega = 600$ rad/s computed at $t = 0.0115$ s.

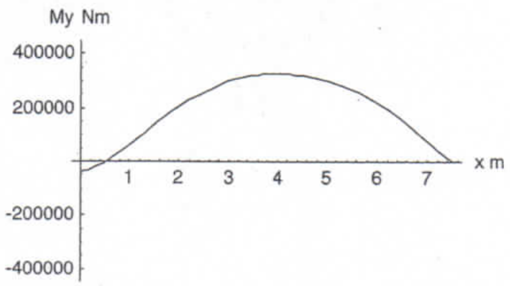

Figure 6. M_y distribution along the x-axis for $\gamma = 0\%$, $\omega = 600$ rad/s computed at $t = 0.0115$ s.

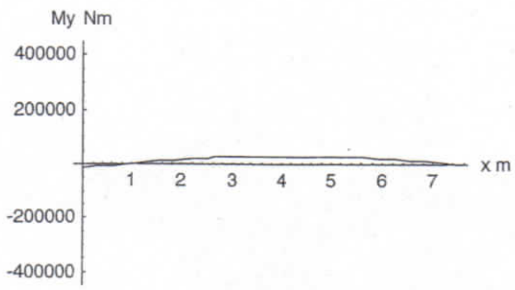

Figure 7. M_y distribution along the x-axis for $\gamma = 10\%$, $\omega = 600$ rad/s computed at $t = 0.0115$ s.

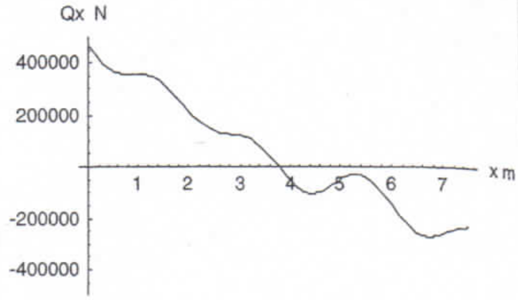

Figure 8. Q_x distribution along the x-axis for $\gamma = 0\%$, $\omega = 600$ rad/s computed at $t = 0.0115$ s.

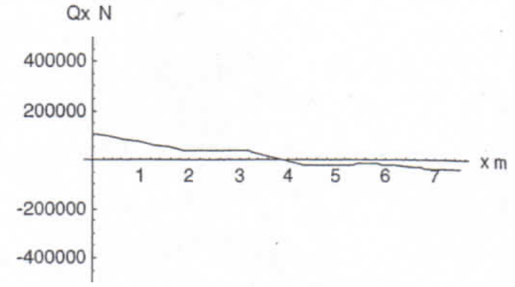

Figure 9. Q_x distribution along the x-axis for $\gamma = 10\%$, $\omega = 600$ rad/s computed at $t = 0.0115$ s.

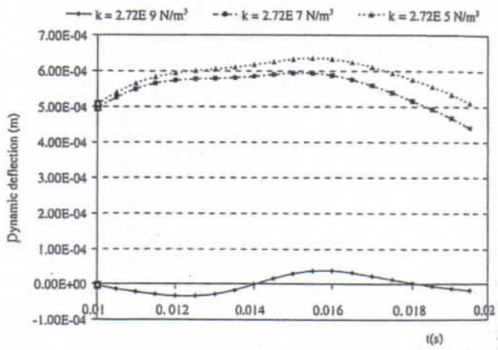

Figure 10. Time history for $\gamma = 0\%$, $\omega = 600$ rad/s for various values of foundation stiffnesses computed at central of the plate.

Figure 10 shows the dynamic deflections of the central point of the plate for different foundation stiffness, but for the same load's frequency and velocity ($\omega = 600$ rad/s and $v = 77$ m/s). It is obvious that when the plate rests on a weak foundation, the dynamic deflection becomes smaller as the foundation stiffness increases. Figures 11–12 show the M_x distribution along the x-axis. It can be seen that by increasing the foundation stiffness by 100 times, the maximum value of M_x decreases drastically.

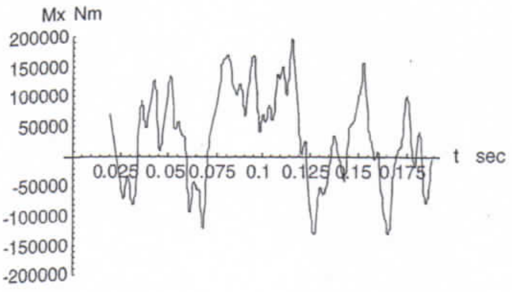

Figure 11. Time history of M_x at central point for $k_f = 2.72 \times 10^8$ N/m^3, $\gamma = 0\%$, $\omega = 600$ rad/s.

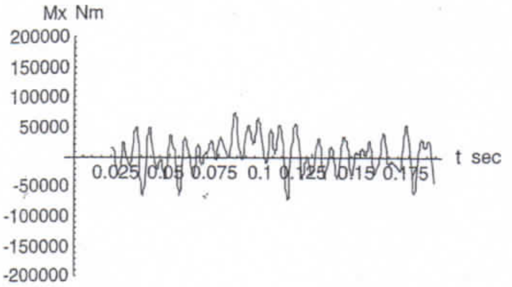

Figure 12. Time history of M_x at central point for $k_f = 2.72 \times 10^{10}$ N/m^3, $\gamma = 0\%$, $\omega = 600$ rad/s.

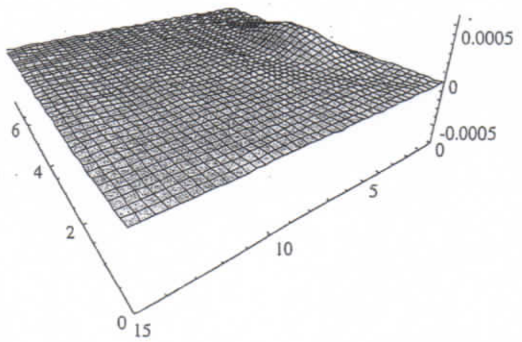

Figure 13. Total dynamic deflection shape calculated at $t = t_0$, $k_f = 2.72 \times 10^{10}$ N/m^3, $\gamma = 0\%$, $\omega = 600$ rad/s.

The effect of the sudden change of position of the dynamic loading on the response is investigated next. Figure 13 shows the shape of the total dynamic deflection at $t = t_0$, when the load suddenly moves to its new position. It can be seen that the deflected area is concentrated only under the dynamic load. Finally, Figure 14 shows the total dynamic load deflection shape at $t > t_0$, when the load continues to move with constant velocity. It can be seen that the deflected area almost covers the entire plate.

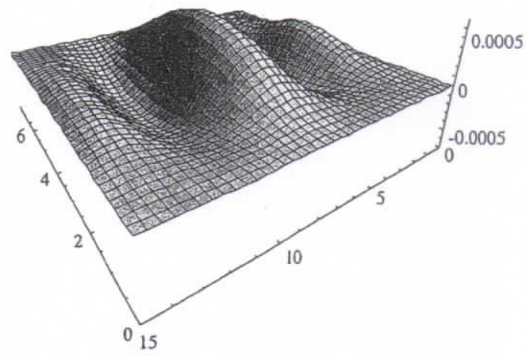

Figure 14. Total dynamic deflection shape calculated at $t > t_0$, $k_f = 2.72 \times 10^{10}$ N/m^3, $\gamma = 0\%$, $\omega = 600$ rad/s.

6 CONCLUSION

In this research work, a procedure is proposed, using the Modified Bolotin Method and the Duhamel integration method to investigate the dynamic behavior of the damped orthotropic plate resting on an elastic Pasternak foundation subjected to dynamic moving loads. The effect of the elastic foundation stiffness, magnitude of the sudden y-position change and damping ratio are discussed. Numerical examples show that the elastic foundation stiffness and damping ratio are the main factors having significant effects on the dynamic response. The type of loading condition affects the dynamic response. When the plate is subjected to a dynamic moving load almost the entire structure is affected.

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