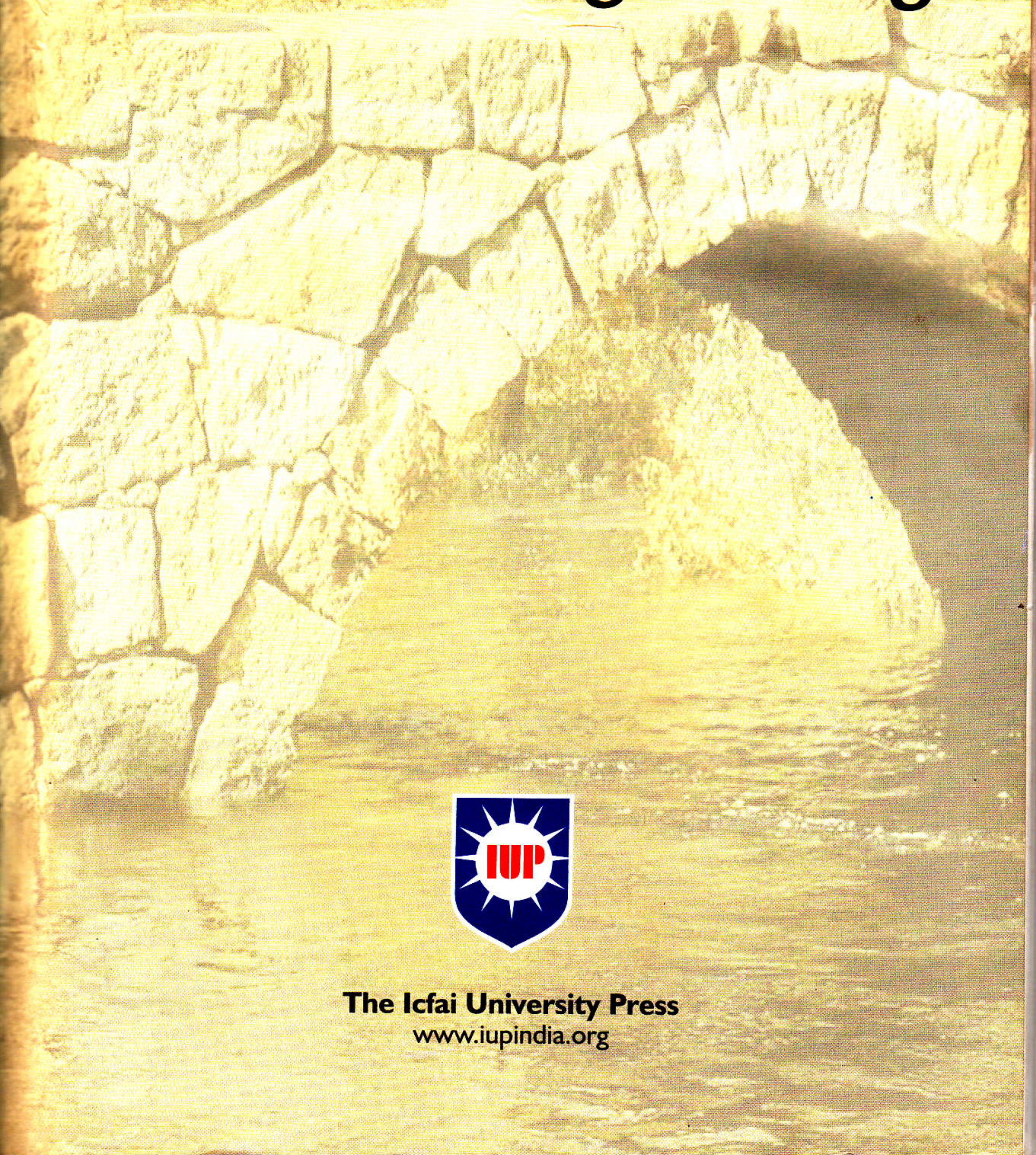
Vol. I No. 1 July 2008

The Icfai University Journal of Structural Engineering



The Icfai University Press
www.iupindia.org

Vol. I No. 1 **July 2008**

The Icfai University Press

52, Nagarjuna Hills, Panjagutta, Hyderabad - 500 082, Andhra Pradesh, India
Tel: +91(40) 23435-368 to 374; Fax: +91(40) 2335-2521
E-mail: info@iupindia.org; Website: www.iupindia.org

Response of Plate on Elastic Foundation Under Harmonic Moving Load

P R Maiti*, Rohit Saha** and Sofia W Alisjahbana***

The vibration of plate rested on foundation to moving load is a problem of great importance in structural dynamics. This paper investigates the dynamic analysis of a finite plate resting on an elastic foundation subjected to traversing point load of harmonic nature, assuming the velocity of load to be constant. The foundation has been modeled as Winkler foundation. Formulations are developed in the transformed field domain using: (1) a double Fourier transform in space; and (2) Laplace Carson integral transform in time domain for steady state response to harmonic moving load. The effect of the speed of the moving load, the foundation stiffness and the dynamic amplification factor are evaluated. The paper also investigates the effect of load frequency on the deflected shapes, and the maximum displacement.

Key Words: Winkler Foundation, Moving Harmonic Load, Velocity, Frequency

Introduction

The investigation of dynamic behavior of plate structures under moving loads has been a topic of interest for well over a century for the design of bridge decks and pavements. Such structures are often subjected to moving loads of high speed vehicles. The design of pavements or decks is traditionally based on the analytical solution of an infinitely long beam or plate under an equivalent static load. Such design methods are deficient as the dimensions of such structure are finite while the moving vehicles exert dynamic load of various amplitudes due to the mechanical vibrations of their engines. Hence, the behavior of structures under moving loads is different from that of static loads. The load amplitude of moving loads is often assumed to be constant. However, the moving loads created by vehicles, in fact, have variations in load amplitude with time that result from the pavement

* Lecturer, Department of Civil Engineering, Institute of Technology, Banaras Hindu University, Varanasi, India. E-mail: pabitra_maiti@yahoo.co.uk

** UG Student, Department of Civil Engineering, Institute of Technology, Banaras Hindu University, Varanasi, India. E-mail: rohit_saha295@rediffmail.com

*** Professor, School of Graduate Studies, Civil Engineering Program, Tarumanagara University, Jakarta, Indonesia. E-mail: wangsadi@indosat.net.id

surface roughness and the mechanical systems of the vehicles. In addition, non-destructive testing vehicles such as rolling dynamic deflectometers apply a steady state harmonic force while continuously moving. Barros and Luco (1992) used moving Green's function to find the response of layered visco-elastic half space to moving and line loads with constant velocity. Zaghloul and White (1993) used a three-dimensional finite element method to find the response of pavement to moving loads. Most of the studies have been conducted for moving loads with constant amplitudes. However, the loads may be harmonic, transient, and random or impact type due to variations in amplitude with time, as a result of many factors like roughness of top surface and mechanical vibrations of the engine of the vehicle (Kim *et al.*, 1995). The dynamic response of plates resting on an elastic foundation has attracted much less attention in comparison to moving loads on decks. The limited investigations involved analytical procedures for plates with simple and regular boundary conditions. Gbadeyan and Oni (1992) gave a closed form solution using double Fourier sine integral transformation to analyze a simply supported rectangular plate resting on an elastic Pasternak foundation traversed by an arbitrary number of moving concentrated masses. Static and free vibration analyses of plates resting on an elastic foundation have been studied extensively, for example, by Saha (1997) and Pevzner *et al.* (2000). Kim and Roesset (1998) investigated an infinite plate resting on an elastic Winkler foundation subjected to moving loads with transformed field domain analyses using Fourier transform. Huang and Thambiratnam (2002) had investigated the dynamic response of plates on elastic foundation subjected to moving loads using the finite strip method and a spring system. In the numerical analysis, the Wilson-θ method was adopted for direct integration. Extensive studies of the dynamic response of plates supported by an elastic foundation with simply supported and unsymmetrical boundary conditions were investigated by Alisjahbana and Wangsadinata (2005 and 2006) using Modified Bolotin Method (MBM). Maiti (2001) solved the plate problem resting on an elastic foundation using Laplace Carson Integral transforms. Maiti and Saha (2006) applied the same technique for solving plate vibration subjected to harmonic loads.

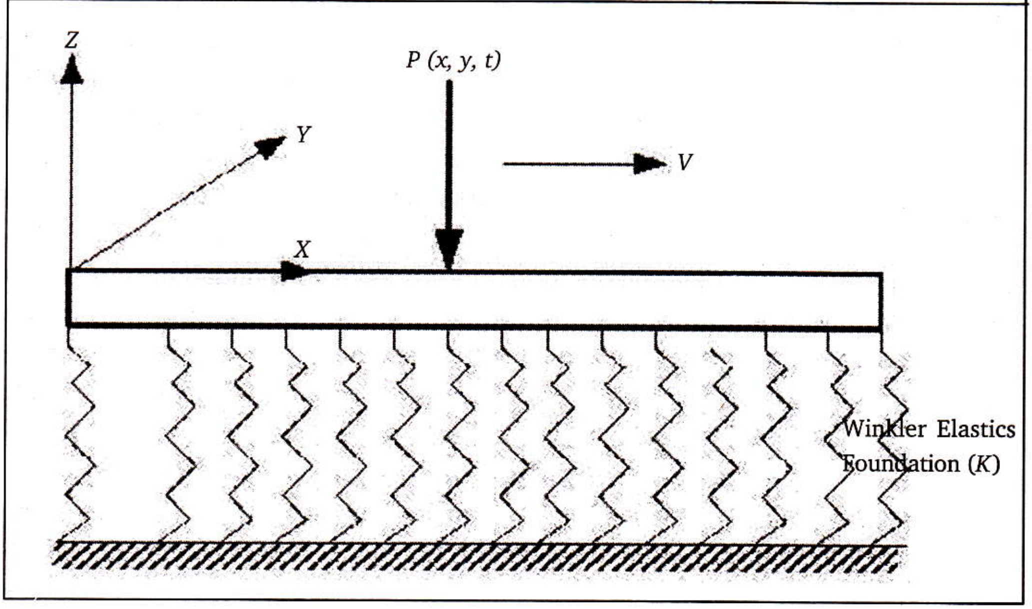
This paper presents an approach that uses Fourier sine integral transforms and Laplace Carson integral transforms to discuss the dynamic response of plate resting on an elastic foundation with moving harmonic concentrated load. Moving loads, in practice, are distributed over a finite area; the point load represents only an idealization. The geometry and material properties were assumed to be linear elastic, and the plate in consideration to be of finite dimensions. The analysis has been carried out in detail for simply supported thin plate with stress-free edges; however, the final formulae with other boundary conditions are also presented.

This technique is flexible and can be easily extended for analysis of plates with various boundary conditions, carrying a moving mass exerting harmonic load. Finally, the results for dynamic response such as deflection, bending and twisting moment, and shear force of the plate are presented with the effects of load velocity and load frequency. In addition, the Dynamic Amplification Factor (DAF) is also discussed.

2. Mathematical Model

We consider a rectangular isotropic plate with a moving mass exerting harmonic load with simply supported edges parallel to X-axis—a moving body with negligible inertia is moving parallel to X-axis with velocity V as shown in Figure 1.

Figure 1: Moving Load on a Plate Rested on Winkler Foundation



The governing differential for a system without damping, neglecting rotatory moments of inertia and shear deformation according to the classical theory of plates can be written as:

$$D\left[\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right]+Kw(x,y,t)$$

$$+\mu\frac{\partial^2 w(x,y,t)}{\partial t^2}=P(x,y,t) \qquad \text{...(1)}$$

In Equation 1, x and y are the Cartesian coordinates and the plate lies on the X-Y plane, w is the lateral deflection of the plate which is a function of x, y and time t, and μ is the mass per unit area of the plate; $D = Eh^3/12(1 - v^2)$ is the flexural stiffness of the plate, where E is the Young's modulus of the plate, and h is the thickness of the plate; $P(x, y, t)$ is the external dynamic load per unit area acting normal to the surface of the plate.

2.1 Boundary Conditions

For simply supported rectangular plate of sides lx and ly, the boundary conditions on w are the initial conditions, i.e., at $t = 0$:

$$w(x,y,0)=0; \frac{\partial w(x,y,t)}{\partial t}=0 \quad \text{...(2)}$$

$$w=0; \frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}=0 \quad \text{at } x=0 \text{ and } x=lx \quad \text{...(3)}$$

$$w=0; \frac{\partial^2 w}{\partial y^2}+v\frac{\partial^2 w}{\partial x^2}=0 \quad \text{at } x=0 \text{ and } y=ly \quad \text{...(4)}$$

2.2 Transformations

For the above boundary conditions, it is convenient to use the two dimensional Fourier sine integral transformations:

$$w(x,y,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{4}{lx\,ly} W(i,j,t) \sin\frac{i\pi x}{lx} \sin\frac{j\pi y}{ly} \quad \text{...(5)}$$

where, $W(i, j, t)$ is the transformation of $w(x, y, t)$. Thus:

$$\int_0^{lx}\int_0^{ly} \frac{\partial^4 w}{\partial x^4} \sin\frac{i\pi x}{lx} \sin\frac{j\pi y}{ly} dx\,dy = \frac{i^4\pi^4}{lx^4} W(i,j,t) \quad \text{...(6)}$$

$$\int_0^{lx}\int_0^{ly} \frac{\partial^4 w}{\partial x^2 \partial y^2} \sin\frac{i\pi x}{lx} \sin\frac{j\pi y}{ly} dx\,dy = \frac{i^2 j^2 \pi^4}{lx^2\, ly^2} W(i,j,t) \quad \text{...(7)}$$

$$\int_0^{lx}\int_0^{ly} \frac{\partial^4 w}{\partial y^4} \sin\frac{i\pi x}{lx} \sin\frac{j\pi y}{ly} dx\,dy = \frac{j^4\pi^4}{ly^4} W(i,j,t) \quad \text{...(8)}$$

The external load $p(x, y, t)$ can be expressed in the form of the Dirac-Delta function:

$$p(x, y, t) = P(t)\delta(x-vt)\delta(x-vt)\delta(y-\eta) \quad ...(9)$$

where, $\delta(x)$ is the Dirac function and $P(t) = a_0 \sin \omega t$

Finally, after the transformation of Equation 1 using Equations 5 to 9, $W(i, j, t)$ can be written as:

$$\ddot{W}(i,j,t) + \omega_{ij}^2 W(i,j,t) = \frac{P(t)}{\mu} \sin \omega_x t \sin \frac{j\pi\eta}{ly} \quad ...(10)$$

where,

$$\ddot{W}(i,j,t) = \frac{\partial^2 W(i,j,t)}{\partial t^2},$$

$$\frac{D}{\mu}\left[\frac{i^4\pi^4}{lx^4} + 2\frac{i^2 j^2 \pi^4}{lx^2\, ly^2} + \frac{j^4\pi^4}{ly^4}\right] + \frac{K}{\mu} = \omega_{ij}^2,$$

$$\frac{i\pi v}{lx} = \omega_x \text{ and}$$

$$P(t) = a_0 \cos \omega t$$

$$\ddot{W}(i,j,t) + \omega_{ij}^2 W(i,j,t) = \frac{a_0}{\mu} \cos \omega t \sin \omega_x t \sin \frac{j\pi\eta}{ly}$$

$$\ddot{W}(i,j,t) + \omega_{ij}^2 W(i,j,t) = \frac{a_0 \sin \frac{j\pi\eta}{ly}}{2\mu} (\sin(\omega_x + \omega)t + \sin(\omega_x - \omega)t)$$

Then, Laplace Carson integral transform is carried out on Equation 10, which can be written as:

$$W^*(i,j,p) = \frac{a_0 \sin \frac{j\pi\eta}{ly}}{2\mu} \frac{1}{p^2 + \omega_{ij}^2} \left[\frac{p(\omega+\omega_x)}{p^2 + (\omega+\omega_x)^2} + \frac{p(\omega_x - \omega)}{p^2 + (\omega_x - \omega)^2}\right] \quad ...(11)$$

where, $W^*(i, j, p)$ is the Laplace Carson transformations of deflection $W(i, j, t)$.

The inverse of Laplace Carson integral transform of Equation 11 can be written as:

$$W(i,j,t) = \frac{1}{2\mu} a_0 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sin\left(\frac{j\pi\eta}{ly}\right) \left[\frac{(\omega+\omega_x)}{\omega_{ij}^2 - (\omega+\omega_x)^2} \left(\frac{\sin(\omega+\omega_x)t}{(\omega+\omega_x)} - \frac{\sin \omega_{ij} t}{\omega_{ij}}\right)\right.$$

$$\left. + \frac{(\omega_x - \omega)}{\omega_{ij}^2 - (\omega_x - \omega)^2} \left(\frac{\sin(\omega_x - \omega)t}{(\omega_x - \omega)} - \frac{\sin \omega_{ij} t}{\omega_{ij}}\right)\right] \quad ...(12)$$

In order to obtain in-field of x and y coordinates, the inverse sine Fourier integral transform of Equation 12 is taken as:

$$w(x,y,t)=\frac{2}{lx\,ly\,\mu}a_0\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sin\left(\frac{j\pi\eta}{ly}\right)\left[\frac{(\omega+\omega_x)}{\omega_{ij}^2-(\omega+\omega_x)^2}\left(\frac{\sin(\omega+\omega_x)t}{(\omega+\omega_x)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right.$$

$$\left.+\frac{(\omega_x-\omega)}{\omega_{ij}^2-(\omega_x-\omega)^2}\left(\frac{\sin(\omega_x-\omega)t}{(\omega_x-\omega)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right]\sin\frac{i\pi x}{lx}\sin\frac{j\pi y}{ly} \quad ...(13)$$

And the corresponding bending moment of the plate:

$$Mx=\frac{2}{lx\,ly\,\mu}a_0\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}D\pi^2\left(\frac{i^2}{lx^2}+\nu\frac{j^2}{ly^2}\right)\sin\left(\frac{j\pi\eta}{ly}\right)\bullet$$

$$\left[\frac{(\omega+\omega_x)}{\omega_{ij}^2-(\omega+\omega_x)^2}\left(\frac{\sin(\omega+\omega_x)t}{(\omega+\omega_x)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right.$$

$$\left.+\frac{(\omega_x-\omega)}{\omega_{ij}^2-(\omega_x-\omega)^2}\left(\frac{\sin(\omega_x-\omega)t}{(\omega_x-\omega)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right]\sin\frac{i\pi x}{lx}\sin\frac{j\pi y}{ly} \quad ...(14)$$

$$My=\frac{2}{lx\,ly\,\mu}a_0\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}D\pi^2\left(\nu\frac{i^2}{lx^2}+\frac{j^2}{ly^2}\right)\sin\left(\frac{j\pi\eta}{ly}\right)\bullet$$

$$\left[\frac{(\omega+\omega_x)}{\omega_{ij}^2-(\omega+\omega_x)^2}\left(\frac{\sin(\omega+\omega_x)t}{(\omega+\omega_x)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right.$$

$$\left.+\frac{(\omega_x-\omega)}{\omega_{ij}^2-(\omega_x-\omega)^2}\left(\frac{\sin(\omega_x-\omega)t}{(\omega_x-\omega)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right]\sin\frac{i\pi x}{lx}\sin\frac{j\pi y}{ly} \quad ...(15)$$

$$Mxy=\frac{2}{lx\,ly\,\mu}a_0\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}D(1-\nu)\frac{ij\pi^2}{lx\,ly}\sin\left(\frac{j\pi\eta}{ly}\right)\bullet$$

$$\left[\frac{(\omega+\omega_x)}{\omega_{ij}^2-(\omega+\omega_x)^2}\left(\frac{\sin(\omega+\omega_x)t}{(\omega+\omega_x)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right.$$

$$\left.+\frac{(\omega_x-\omega)}{\omega_{ij}^2-(\omega_x-\omega)^2}\left(\frac{\sin(\omega_x-\omega)t}{(\omega_x-\omega)}-\frac{\sin\omega_{ij}t}{\omega_{ij}}\right)\right]\cos\frac{i\pi x}{lx}\cos\frac{j\pi y}{ly} \quad ...(16)$$

3. Numerical Example

Let us consider a finite length plate rested on elastic foundation with the soil stiffness value $k = 3.5 \times 10^6$ N/m^3, plate length lx = 100 m and breadth ly = 20 m, thickness h = 0.3 m, ν = 0.25, m = 355 kg/m^2, $E = 3.1 \times 10^{10}$ N/m^2. The DAF is the ratio of the displacement of the plate with the maximum static displacement for a particular loading and plate dimension.

The dynamic displacement response of the plate rested on elastic foundation, when subjected to moving load with harmonic amplitude variation, is investigated. The maximum displacements under a moving harmonic load are examined for various values of the velocity and load frequency.

Figure 2 shows the displacement response of the plate when a moving load of magnitude 10 kN is moving with a constant velocity of 20 m/s through the plate. It is observed that maximum deflection and bending moment occur when the load reaches the central point of the plate (Figures 3-5).

Figure 6 shows that the displacement of the plate changes with the change of foundation stiffness. As the stiffness of the foundation increases the displacement response decreases. From Figure 7, it is observed that on increasing the load frequency and load velocity, the displacement of the plate increases.

Figure 2: Displacement History Graph at Central Point for P_0=10 kN and ν = 20 m/s

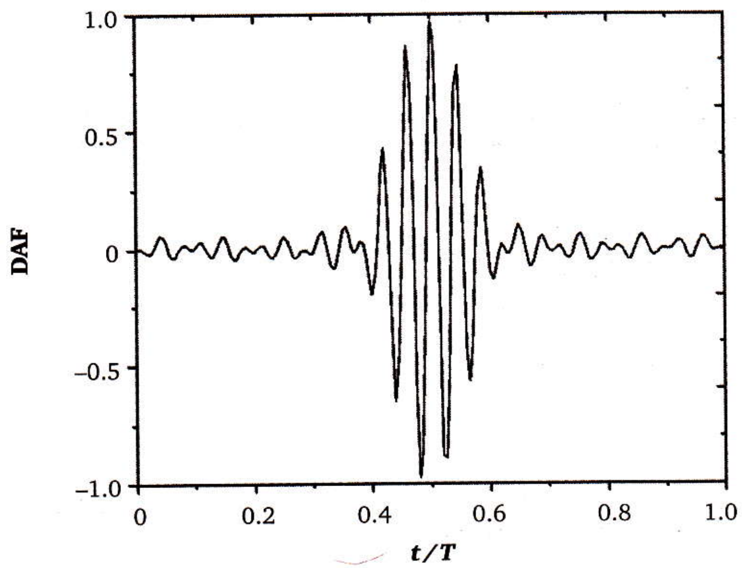


Figure 3: Moment History Graph at Central Point Along X-Direction

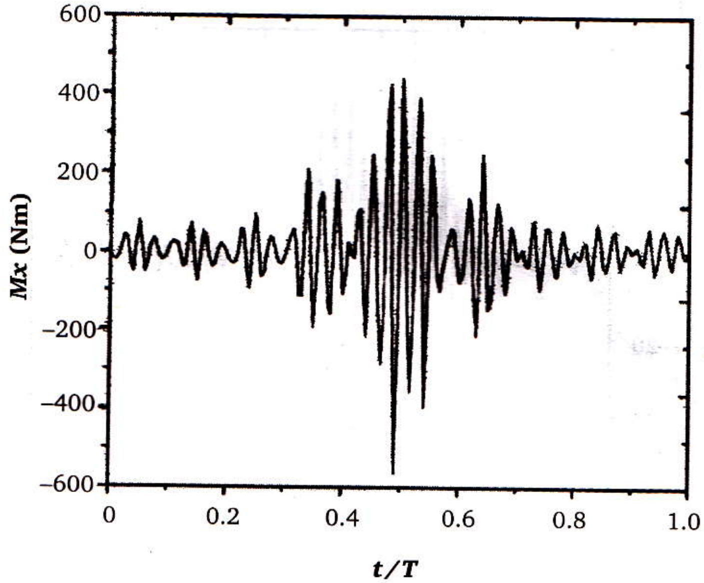


Figure 4: Displacement History of Bending Moment, *My*, at Central Point

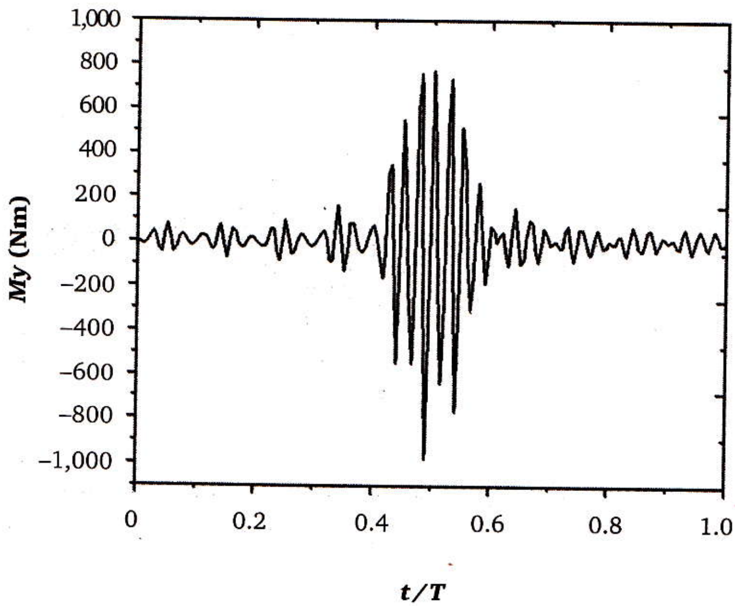


Figure 5: Twisting Moment History of Plate Computed at Position $x = 40$ m and $y = 15$ m

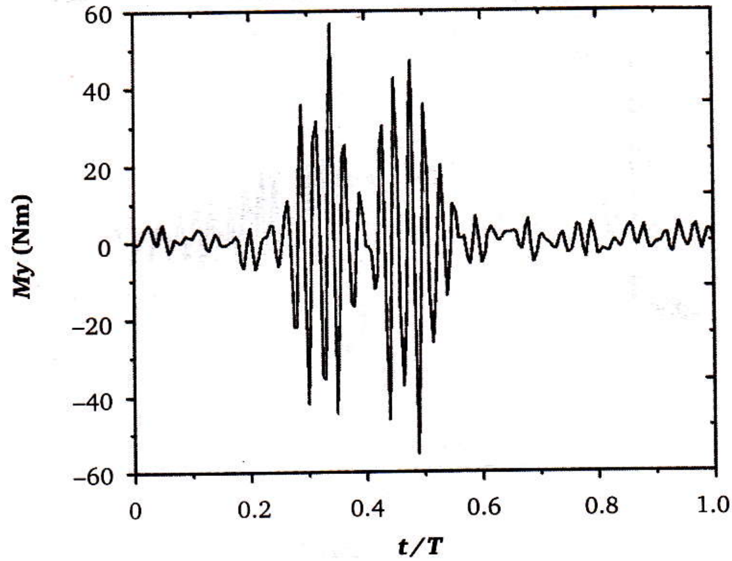


Figure 6: Dynamic Amplification Factor History for Different Values of Foundation Stiffness

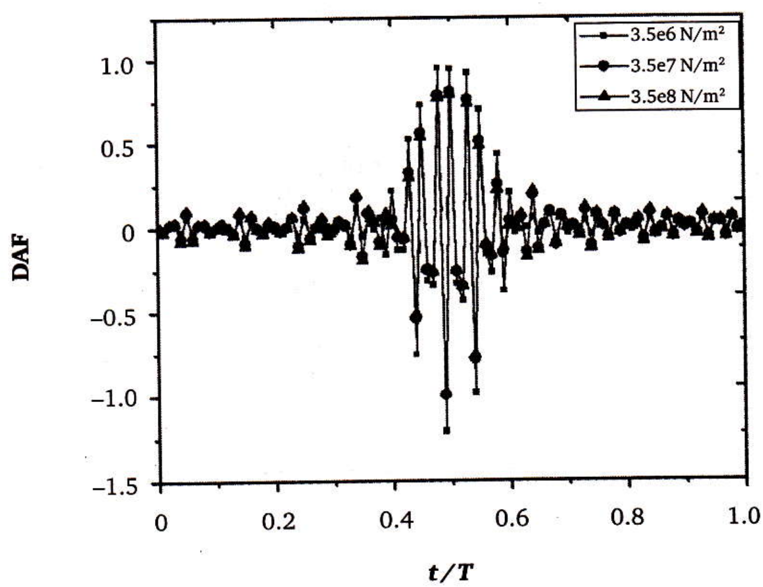
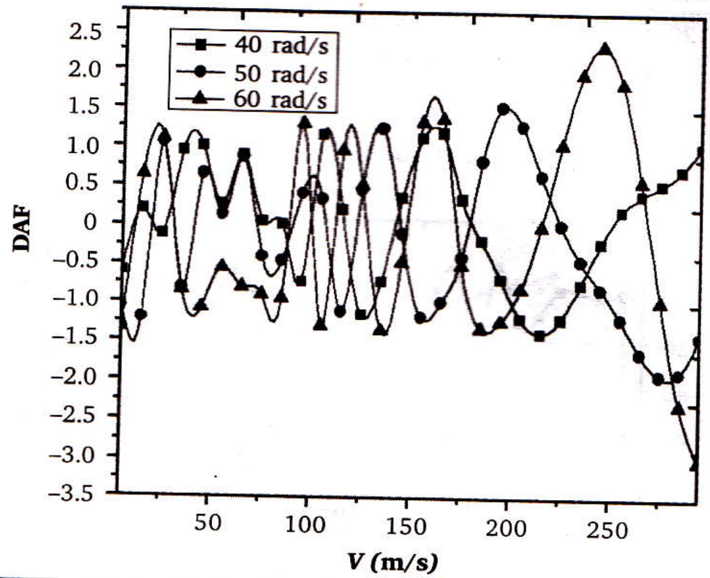


Figure 7: Dynamic Amplification Factor for Different Load Frequencies



In Figure 8, *Mx* and *My* is plotted for different values of load velocity (*V*). It is observed that the moment in the *y*-direction (*My*) is higher than the moment in the *x*-direction (*Mx*), and both the moments increase as the load velocity increases. Figure 9 shows that twisting moment increases slowly at lower load velocities and at a very high rate at higher load velocities.

Figure 8: *Mx* and *My* as a Function of Load Velocity (*V*)

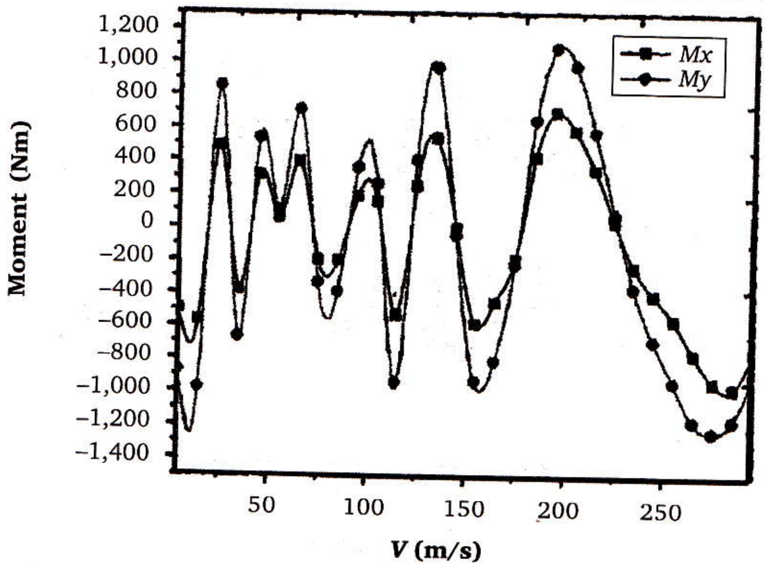
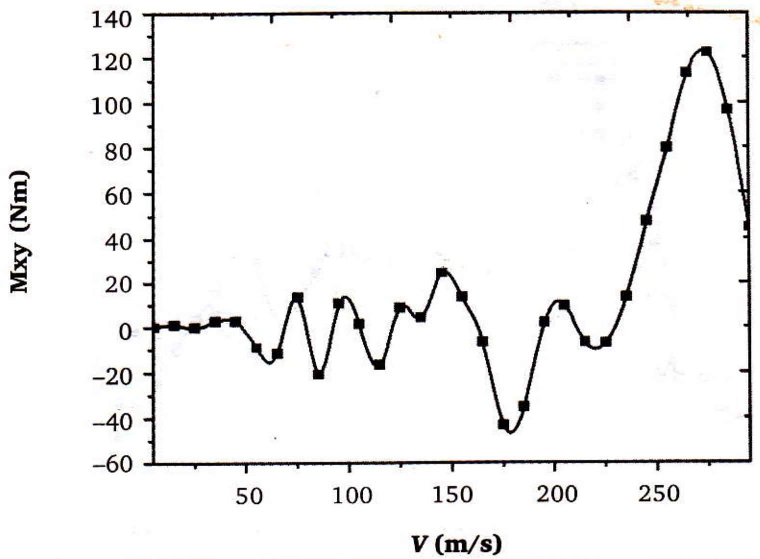
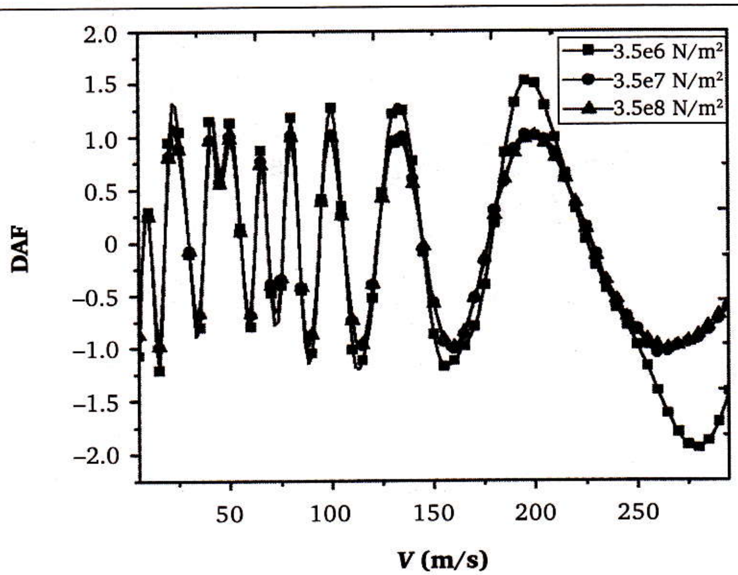


Figure 9: Twisting Moment (*Mxy*) as a Function of Load Velocity (V)



The DAF for displacement of the plate is harmonic in nature as the loading applied is harmonic. From Figure 10, it is observed that the variation of DAF increases with increasing load. There is some critical velocity at which maximum deflection occurs due to moving load. Here, it is observed that maximum deflection occurs at around the load velocity, 300 m/s.

Figure 10: DAF as a Function of Load Velocity for Different Values of Foundation Stiffness



In Figure 11, DAF has been plotted for lower load velocities of 10 m/s, 20 m/s and 40 m/s with different loading frequencies. The variation of load frequency is changing the dynamic response with same gradient. Figure 12 shows that the DAF for higher load velocities becomes periodic in nature, with increasing the load frequency.

Figure 11: DAF as a Function of Load Frequency at Lower Velocities

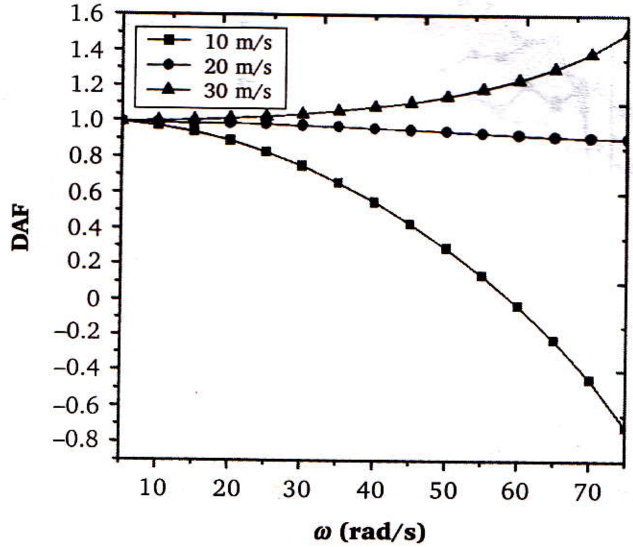
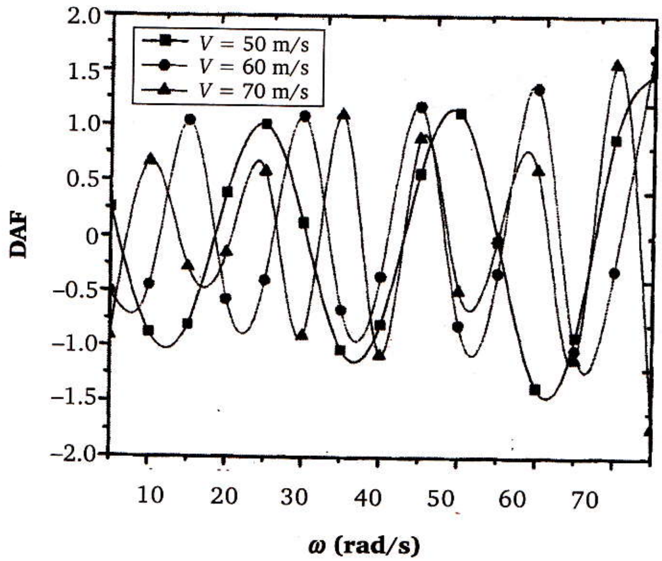


Figure 12: DAF as a Function of Load Frequency Computed at High Velocity Values



In Figures 13 and 14, DAF has been plotted for different soil stiffness and load velocity, respectively, along the plate length. The variation of moment *Mx*, *My* and *Mxy* along the plate length has been plotted in Figures 15 and 16. The deflected

Figure13: Variation of DAF Along the Plate Length for Different Values of Foundation Stiffness

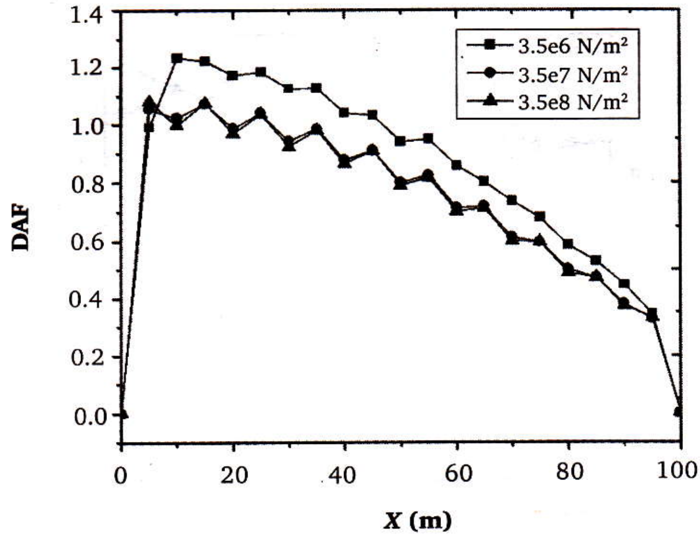


Figure14: Variation of DAF Along the Longitudinal Direction of Plate for Different Values of Velocity

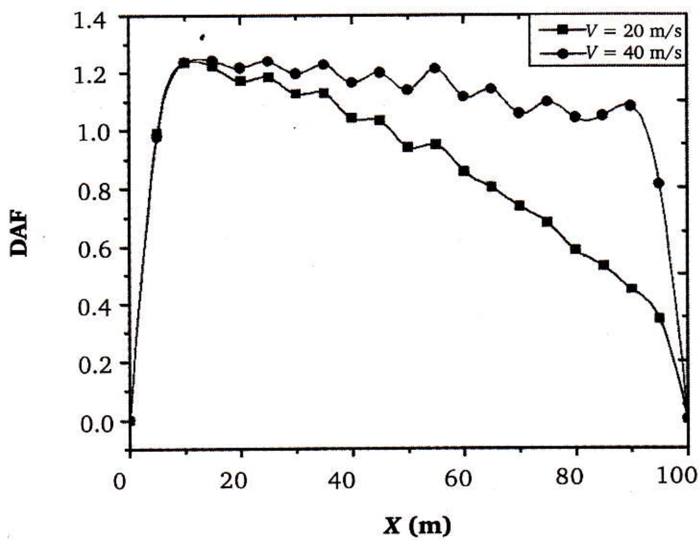


Figure 15: Variation of Central Point Moment Along Longitudinal Direction

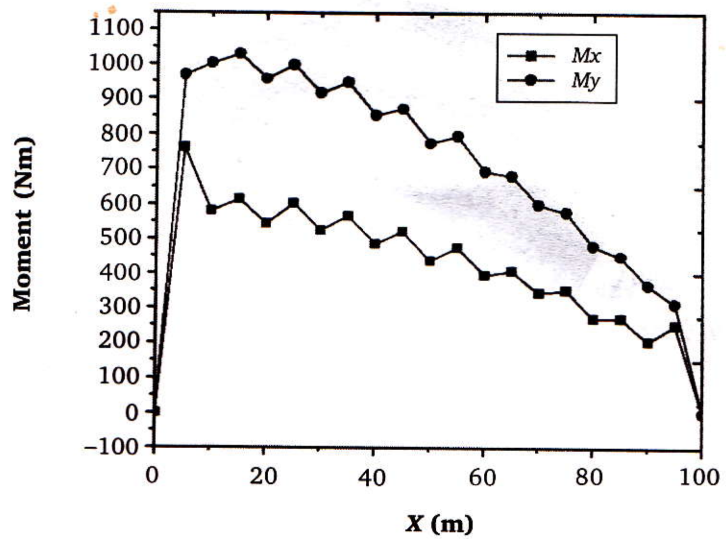
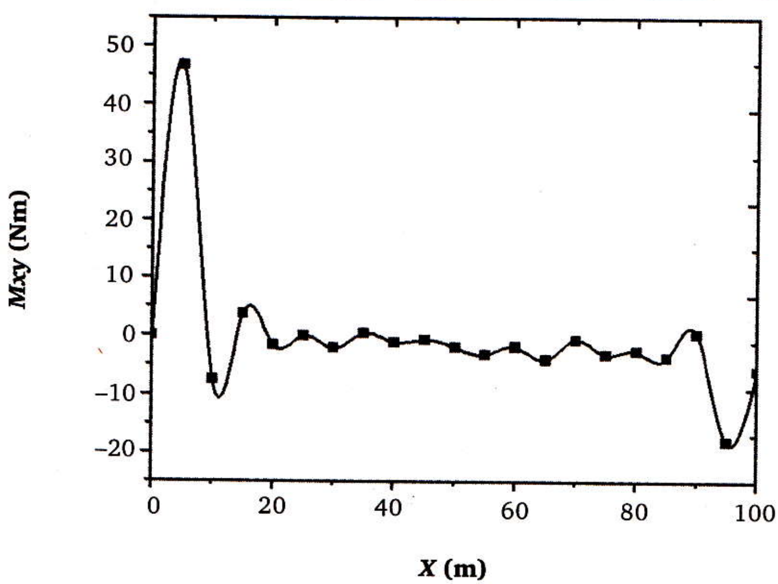
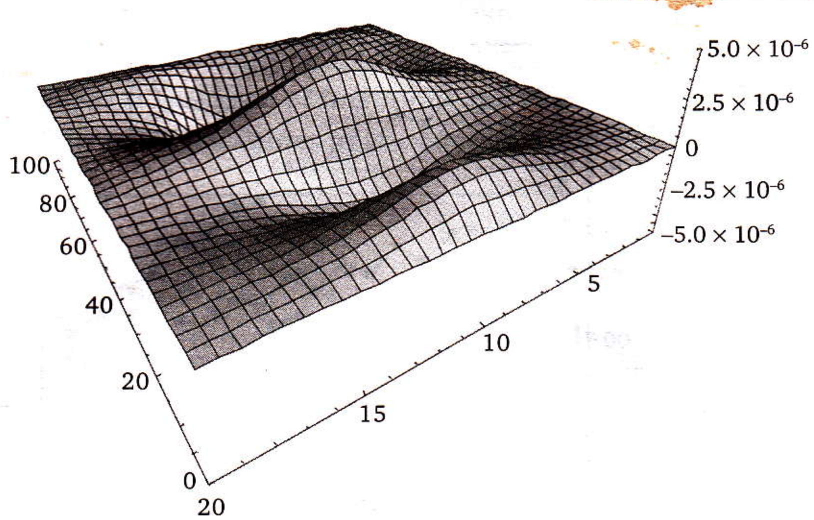


Figure 16: Variation of Twisting Moment Along Longitudinal Direction



shape of the entire plate at a particular load velocity, 50 m/s, and load frequency, 50 rad/s, has been plotted in Figure 17.

Figure17: Total Deflected Shape of the Plate Subjected to a Moving Load Computed at $t = 2.5$ s



Conclusion

The dynamic response of a finite plate rested on Winkler foundation subjected to moving harmonic load has been investigated using sine Fourier integral transformation in space, and Laplace Carson integral transform in time domain. The formulation neglected the effects of rotatory inertia and transverse shear deformation; it cannot produce surface waves propagating along the plate. The DAF of the plate is determined for different loading conditions and load velocities. The results show that the effect of foundation stiffness and load velocity is an important parameter for deflection of the plate under dynamic conditions.

References

1. Alisjahbana S W and Wangsadinata W (2005), "Dynamic Response of Clamped Orthotropic Plates to Dynamic Moving Loads", The 3rd Int. Conference on Construction Materials: Performance, Innovations and Structural Implications, Vancouver B C, Canada, August 22-24.

2. Alisjahbana S W and Wangsadinata W (2006), "Response Dynamics of Rigid Runway Pavements", Proceedings of Tenth East Asia Pacific Conference on Structural Engineering and Construction, Wind and Earthquake Engineering, pp. 375-380, August 3-5, Bangkok, Thailand.

3. Barros F C P and Luco J E (1992), "Moving Green's Functions for layered Viscoelastic Half Space", Department of Applied Mechanics and Engineering Sciences, Rep., Univ. of California, San Diego, California.

4. Gbadeyan J A and Oni S T (1992), "Dynamic Response to Moving Concentrated Masses of Elastic Plates on a Non-Winkler Elastic Foundation", *Journal of Sound and Vibration*, Vol. 154, No. 2, pp. 343-358.

5. Huang M H and Thambiratnam D P (2002), "Dynamic Response of Plates on Elastic Foundation to Moving Loads", *ASCE Journal of Eng. Mechanics*, pp. 1016-1022.

6. Kim S M and Roesset J M (1998), "Moving Loads on a Plate on Elastic Foundation", *ASCE Journal of Engineering Mechanics*, Vol. 124, No. 9, pp. 1010-1017.

7. Kim S M, Roesset J M, White T D and Hugo F (1995), "Dimensional Analysis of Load Simulator Action on Pavements", Center of Transportation Research, Rep. 2914-1F, University of Texas at Austin, Austin, Texas.

8. Maiti P R (2001), "Dynamic Response of Plates on Elastic Foundation Under Moving Load", M E Thesis submitted at Bengal Engineering and science University, Shibpur, WB, India.

9. Maiti P R and Saha R (2006), "Dynamic Behavior of Plates on Elastic Foundation Under Moving Harmonic Load", Proceedings of Tenth East Asia Pacific Conference on Structural Engineering and Construction, Analytical and Computational Methods, pp. 95-100, August 3-5, Bangkok, Thailand,

10. Pevzner P (2000), "Further Modification of Bolotin Method in Vibration Analysis of Rectangular Plates", *AIAA Journal*, Vol. 38, No. 9, pp. 1725-1729.

11. Saha K N (1997), "Dynamic Stability of a Rectangular Plate on Non-Homogeneous Winkler Foundation", *Computers and Structures*, Vol. 63, No. 6, pp. 1213-1222.

12. Zaghloul S M and White T D (1993), "Use of Three Dimensional Dynamic Finite Element Model Program for Analysis of Flexible Pavement", *Transp. Res. Rec.*, National Research Council, Vol. 1388, pp. 60-69.

Reference # 62J-2008-07-05-01