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RESPONSE OF CLAMPED ORTHOTROPIC PLATE SUBJECTED TO THE DYNAMIC MOVING LOAD

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Abstract

An approximate method is presented for determination of the natural frequencies and mode shapes of a rectangular orthotropic plate with all sides clamped. The natural frequencies of a clamped plate are presented in a form analogous to the corresponding frequencies of a simply supported plate. The wave numbers are the unknown quantities that can be determined from a system of two transcendental equations obtained from the solution of two auxiliary Levy's type problems. The general solution for the forced response is given in an integral form based on Duhamel's method with several useful examples included.

Keywords natural frequency, orthotropic, transcendental equations, forced response

1. Introduction

Widespread used of filamentary composite materials in several fields of modern technology has made it desirable to investigate the dynamic behavior of structures under the effects of material anisotrophy. Analytical and experimental studies of small deflection free vibration of orthotropic plates have been made by many authors. The most comprehensive study was done by Leissa [5]. An exact solution of the differential equation of a vibrating orthotropic plate is known for the case of rectangular plate simply supported along one pair of opposite edges, which is also known as Levy's problem. The exact solution for the plate with all sides clamped is unknown. At the same time a considerable number of approximate solutions are available in the literature for several combinations of boundary conditions, including the case of clamped plates. Elishakoff in 1974 [4] investigated the dynamic analysis of a clamped square orthotropic plates. As a point of departure in his analysis, the presentation of the frequencies in a form fully analogous to the corresponding frequency of simply supported plate is used. For a simply supported plate, the wave number are equal to $m\pi/a$ and $n\pi/a$ respectively, where a denotes the length of the square's side and m and n are positive integers which determine the number of mode shape.

The present analysis deals with the dynamic response of a rectangular clamped orthotropic plate subjected to the dynamic transverse moving load. In this analysis, the wave numbers are adopted from the worked done by Elishakoff in 1974. These wave numbers are presented in the form of $p\pi/a$ and $q\pi/b$, where the pair of real quantities p and q are to be found from the solution of two supplementary eigenvalue problems. The integer parts of p and q represent frequency numbers. The mode shape is presented as a product of eigenfunctions. The dynamic solution of the plate is based on orthogonality conditions of eigenfunctions similar to those used by Alisjahbana, S.W. [2] in analyzing forced response of simply supported rectangular orthotropic plates subjected to the dynamic transverse moving load. The dynamic response of the plate can be expressed in integral form that can be readily integrated to determine the plate response for any applied surface loading $p(x,y,t)$.

where n denotes the direction normal to the contour of the plate.

For the plate with all sides simply supported that satisfies Equation (5), it is seen that $W(x,y)$ can be expressed as

$$W_{mn}(x,y) = A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{7}$$

where A_{mn} is an amplitude coefficient determined from the initial conditions of the problem and m and n are positive integers. Substituting Equation (7) into Equation (3) gives the natural frequency of the system

$$\omega_{mn} = \left(\frac{B}{\rho h}\right)^{1/2} \left[\frac{D_x}{B}\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \frac{D_y}{B}\left(\frac{n\pi}{b}\right)^4\right]^{1/2} \tag{8}$$

The purpose of this paper is to find the solution of Equation (3) with the boundary conditions (6), i.e. the eigen frequencies and the mode shapes of a clamped orthotropic plate. By postulating the following eigen frequency which is analogous to the case of a plate with all sides simply supported (Elishakoff, I.B 1974), Equation (8) can be expressed as

$$\omega_{mn} = \left(\frac{B}{\rho h}\right)^{1/2} \left[\frac{D_x}{B}\left(\frac{p\pi}{a}\right)^4 + 2\left(\frac{p\pi}{a}\right)^2\left(\frac{q\pi}{b}\right)^2 + \frac{D_y}{B}\left(\frac{q\pi}{b}\right)^4\right]^{1/2} \tag{9}$$

where p and q are real numbers which are to be found.

These two real numbers p and q can be determined by considering two auxiliary problems of Levy's type (Elishakoff, I.B 1974).

2a. First Auxiliary Problem

The solution of Equation (4) for the first auxiliary problem that satisfied the boundary conditions

$$X(x) = \frac{dX(x)}{dx} = 0 \quad \text{at } x=0,a \tag{10}$$

can be expressed as

$$W(x,y) = X(x)\sin\frac{q\pi y}{b} \tag{11}$$

Substituting Equation (11) into Equation (4) results in an ordinary differential equation for X(x)

$$\frac{d^4X(x)}{dx^4} - 2\frac{q^2\pi^2}{b^2}\frac{B}{D_x}\frac{d^2X(x)}{dx^2} + \left(\frac{q^4\pi^4}{b^4}\frac{D_y}{D_x} - \frac{\rho h\omega^2}{D_x}\right)X(x) = 0 \tag{12}$$

or, given the postulating Equation (9), one obtains

$$\frac{d^4X}{dx^4} - 2\frac{q^2\pi^2}{b^2}\frac{B}{D_x}\frac{d^2X}{dx^2} - \frac{p^2\pi^2}{a^2}\left[2\frac{B}{D_x}\frac{q^2\pi^2}{b^2} + \frac{p^2\pi^2}{a^2}\right]X = 0 \tag{13}$$

The corresponding characteristic equation

$$\lambda^4 - 2\frac{q^2\pi^2}{b^2}\frac{B}{D_x}\lambda^2 - \frac{p^2\pi^2}{a^2}\left[2\frac{B}{D_x}\frac{q^2\pi^2}{b^2} + \frac{p^2\pi^2}{a^2}\right] = 0 \tag{14}$$

Equation (14) has two imaginary and two real roots, namely

$$\lambda_{1,2} = \pm\frac{\pi}{ab}k_1, \quad \lambda_{3,4} = \pm i\frac{p\pi}{a} \tag{15}$$

$$\lambda_{1,2} = \pm\frac{\pi}{ab}\kappa_2 \qquad \lambda_{3,4} = \pm i\frac{q\pi}{b} \tag{24}$$

where

$$\kappa_2 = \sqrt{\frac{2p^2b^2B}{D_y} + q^2a^2} \tag{25}$$

The general solution of second auxiliary problem can be represented as

$$Y(y) = B_1\cosh(\kappa_2\pi\eta_2) + B_2\sinh(\kappa_2\pi\eta_2) + B_3\cos(q\pi\eta_2 a) + B_4\sin(q\pi\eta_2 a) \tag{26}$$

where B_i are constants of integration and $\eta_2 = \frac{y}{ab}$.

By substituting Equation (26) into Equation (23), one obtains the following characteristics determinant

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & \frac{\kappa_2}{a} & 0 & q \\ C_2 & S_2 & c_2 & s_2 \\ \frac{\kappa_2}{a}S_2 & \frac{\kappa_2}{a}C_2 & -qs_2 & qc_2 \end{vmatrix} = 0 \tag{27}$$

where

$$C_2 = \cosh\left(\frac{\kappa_2\pi}{a}\right) \qquad c_2 = \cos(q\pi)$$
$$S_2 = \sinh\left(\frac{\kappa_2\pi}{a}\right) \qquad s_2 = \sin(q\pi) \tag{28}$$

Expanding Equation (27) and substituting Equation (24) into the result gives

$$1 - c_2C_2 + \frac{B}{D_y}\left[2\frac{B}{D_y}\frac{q^2a^2}{p^2b^2} + \frac{q^4a^4}{p^4b^4}\right]^{\frac{1}{2}} s_2S_2 = 0 \tag{29}$$

3. Frequencies and Mode Shapes

Equations (20) and (29) being transcendental in nature, have an infinite number of roots. The unknown quantities p and q are calculated from the solution of these equations. By substituting p and q into Equation (9), one obtains the eigen frequencies. The integer parts p and q represent the numbers of the eigen frequency. The mode shapes are determined as the product

$$W_{pq}(x,y) = X(x)Y(y) \tag{30}$$

where

$$X(x) = \cosh(\kappa_1\pi\eta_1) - \cos(p\pi\eta_1 b) + \left[\frac{c_1 - C_1}{S_1 - \frac{\kappa_1}{bp}s_1}\right]\left[\sinh(\kappa_1\pi\eta_1) - \frac{\kappa_1}{bp}\sin(p\pi\eta_1 b)\right] \tag{31}$$

and

$$
\begin{aligned}
w(x,y,t) &= \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\left[X_m(x)Y_n(y)e^{-\bar{\gamma}\omega_{mn}t}\left(a_{mn}\cos\left(\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}t\right)-b_{mn}\sin\left(\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}t\right)\right)\right] \\
&+\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\left[X_m(x)Y_n(y)P_0X_m(x_0)Y_n(y_0)\frac{1}{\rho h Q_{mn}\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}}\right. \\
&\left.\int_0^t\left[\cos(\omega\tau)e^{-\bar{\gamma}\omega_{mn}(t-\tau)}\sin\left(\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}(t-\tau)\right)\right]d\tau\right]
\end{aligned}
\tag{37}
$$

For $t>t_0$:

$$
\begin{aligned}
w(x,y,t) &= \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\left[X_m(x)Y_n(y)e^{-\bar{\gamma}\omega_{mn}t}\left(a_{mn}\cos\left(\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}t\right)-b_{mn}\sin\left(\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}t\right)\right)\right] \\
&+\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\left[X_m(x)Y_n(y)P_0X_m(x_2)Y_n(y_0)\frac{1}{\rho h Q_{mn}\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}}\right. \\
&\left.\int_0^t\left[\cos(\omega\tau)e^{-\bar{\gamma}\omega_{mn}(t-\tau)}\sin\left(\sqrt{1-\bar{\gamma}^2}\,\omega_{mn}(t-\tau)\right)\right]d\tau\right]
\end{aligned}
\tag{38}
$$

This problem may be treated in two parts. The first part involves a harmonically oscillating concentrated transverse load at a constant position (x_0,y_0). The second part involves a harmonically oscillating concentrated transverse load at a constant position (x_2,y_0). The two parts of the problem are related through the initial conditions. The motion of the plate at $t=t_0$ due to the load at (x_0,y_0) becomes the initial condition of the plate subsequent instantaneous change at $t=t_0$, due to the load at (x_2,y_0).

Using the above analysis, the motion during an interval of time including the sudden change of the load position in x direction can be computed.

6. Numerical Example

Using the procedure of the last section, some results have been obtained for a rectangular clamped orthotropic plate with a dynamic moving load. Three types of phenomena are presented affecting the total dynamic deflection: the effect of changing the load's frequency ω, the effect of changing the damping ratio ($\bar{\gamma}$), three cases have been calculated for which $\bar{\gamma}$ equaled 0, 0.03 and 0.1, representing the damping factors for building structures. The position of the dynamic moving load is varied from x_0=2m to x_0=2.75m. The weight of the dynamic transverse load is P_0=1000N and t_0=10 sec, being the instant when the dynamic deflection of the plate is near its maximum.

The following numerical results have been calculated for the case of a thin rectangular clamped orthotropic plate with dimensions and characteristics as follows: a=15m, b=20m, ρ=2400kg/m^3, h=0.12m, E_x=30x10^{9}N/m^2, E_y=20x10^{9}N/m^2, υ_x=0.2, υ_y=0.1, G=10^{10}N/m^2.

Based on the data shown above, the natural frequencies of the plate for the first 4 modes in x direction (m=1,2,...,4) and the first 5 modes in y direction (n=1,2,...,5) are calculated.

Figure 2 shows the motion at mid-span for the case of harmonically concentrated transverse load with a load frequency of 20 rad/sec. The value of t_0 is chosen such that it occurs when the dynamic deflection of the system is near a maximum. Note in this figure that at $t=t_0$, a transient motion begins due to the load at $x=x_2$.

Figure 3 shows the maximum deflection response spectra at the mid-span of an orthotropic plate. Note in this figure that at the value of the load's frequency approaching the natural frequency of the system, the dynamic deflection increases for all values of damping ratio. Damping plays its useful role of reducing the maximum dynamic deflection. Analysis of this nature is particularly useful for applications where allowable maximum dynamic deflections are a constraint.

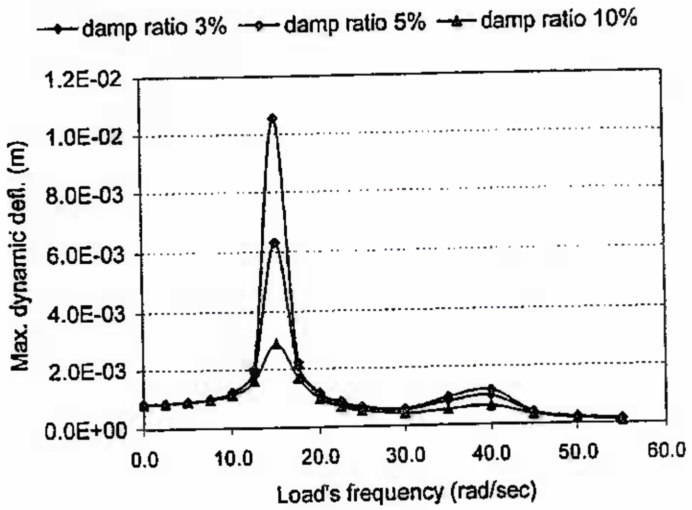

Figure 3. Maximum dynamic deflection response spectra for various values of load's frequency. Load's parameter: P_0=1000N, t_0=10sec, $\bar{\gamma}$=3%, x_0=5m, y_0=5m, x_1=0.5m.

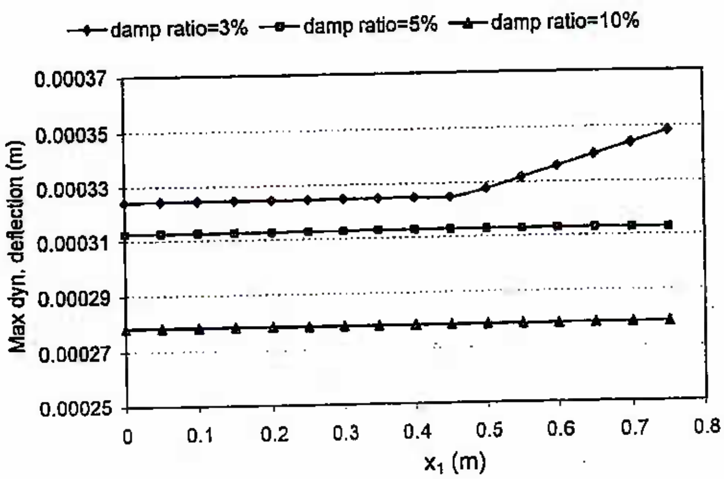

Figure 4. Maximum dynamic deflection response spectra for various values x_1. Load's parameter: P_0=1000N, ω=20 rad/sec, t_0=10sec, $\bar{\gamma}$=3%, x_0=2m, y_0=5m, x_1=0.5m.

Figure 4 shows the maximum dynamic deflection at mid-span of the orthotropic plate due to the sudden change in x position of a concentrated load of varying amplitude as a function of the magnitude of the sudden in x position, x_1, and of the damping ratio, $\bar{\gamma}$. From the figure, it is obvious that the maximum dynamic deflection is a function of the magnitude of the sudden change in the x position. As expected, the maximum dynamic deflection increases as the change in the x position increases for all values of damping ratio.

Figure 5 shows examples of the total dynamic deflection mode shapes of an orthotropic rectangular plate clamped on all sides during the interval of $t<t_0$ for m=1,2,...,4 and n=1,2,...,5.