Response of Damped Orthotropic Stiffened Plates Subjected to a Stepped Triangular Blast Loading

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Abstract

The response of structural components subjected to a stepped triangular blast loading had been the subject of a great deal of research. Much of these researches had been concentrated on the response of plates to impulsive loads. Complexity of these problem would increases appreciably when effects due to stiffeners configuration and effects of structural damping were considered. In this paper the behavior of orthotropic damped plates with different stiffener configurations subjected to a stepped triangular blast loading is described. The aim of this work is to determine the dynamic response of stiffened orthotropic plates to a stepped trianggular blast loading and to determine the effect of stiffeners configuration and blast loading duration on the plate failure. The natural frequencies of the orthotropic plate were solved from two transcendental equations, while the eigen functions of the system are solved using the Modified Bolotin Method (MBM). The stepped traingular blast loading is further integrated using the Duhamel integration method to find the total dynamic response of the system. The numerical results give new insight into the effect of stiffeners configuration, structural damping and time duration of the blast loading on the plate response.

Keywords: Stepped triangular blast loading, orthotropic plate, stiffeners, Modified Bolotin Method, time duration

1. INTRODUCTION

The dynamic response of damped orthotropic stiffened plates subjected to blast and impact loading had been studied quite extensively in recent years. To provide adequate protection against explosions, the

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design and construction of public buildings such as hospitals, school, hotel have received renewed attention of structural engineers.

Dobyns (Dobyns 1981) had analyzed a simply supported orthotropic plate subjected to static and dynamic loads, which included the numerical solution of a plate subjected to blast loads that were modeled as a triangular function, an exponential function and a stepped triangular function. Kadid (Kadid et al. 2007) had presented a numerical study of stiffened plates subjected to uniform blast loading, which included the effect of the stiffeners configurations upon the plate's response. However, most of the work available in the published literature is dealing with plates with simply supported boundary conditions. This is because the dynamic transverse displacement can be represented by terms of a Fourier series. Previous extensive studies on the dynamic response of stiffened orthotropic plate subjected to a constant triangular blast loading with a very general restraint condition along its supports had been conducted by Alisjahbana and Wangsadinata (Alisjahbana and Wangsadinata 2009). The previous work is extended in this paper to examine the response of stiffened orthotropic plate under localized blast loading, which is modeled as a stepped triangular blast loading, whereby the plate is fully fixed along its supports.

The geometry and material properties are assumed to be linear elastic and the orthotropic stiffened plate under consideration is of finite dimensions. The numerical results presented in this paper can provide better understanding of the behavior of plate structures under blast loading, while explosive tests are costly and dangerous without any assurance of the certainty of the results.

2.   VIBRATION ANALYSIS

In the first part of this paper the free vibration of orthotropic stiffened plate with fully fixed condition along its support is studied first using the Levy's solution. The free vibration solution of the system is set as:

$$w(x, y, t) = W(x, y) \sin \omega t \tag{1}$$

where $W(x,y)$ is a function of the position coordinates only, and   is the circular frequency.

The undamped free vibration equation of motion of the system can be expressed as:

$$D_x \frac{\partial^4 W(x, y)}{\partial x^4} + 2B \frac{\partial^4 W(x, y)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W(x, y)}{\partial y^4} - \rho h \omega^2 W(x, y) = 0 \tag{2}$$

where D_x and D_y are the flexural rigidities in x and y direction respectively, B is the torsional rigidity, γ is the damping ratio, ρ is the mass density of the plate. The plate is stiffened by stiffeners of width b_x and height h_x. The origin of the Cartesian coordinates (x,y) is set at the upper left corner of the plate.

The boundary conditions of the orthotropic stiffened plate for fully fixed condition along its support are as follow:

Along $x=0$ and $x=a$:

$$W(x, y) = \frac{\partial W(x, y)}{\partial x} = 0 \tag{3}$$

Along $y=0$ and $y=b$:

$$W(x, y) = \frac{\partial W(x, y)}{\partial y} = 0 \tag{4}$$

The next step is to find the solution of the free vibration equation (2) with the boundary condition according to equation (3) and equation (4) to obtain the Eigen frequencies and the mode shapes of the orthotropic plate. By postulating the Eigen frequencies to be analogous to the case of a plate with simply supported condition at all edges (Pevzner et al. 2000), natural frequencies of the system can be expressed as:

$$\omega_{mn} = \sqrt{\frac{\pi^4}{\rho h}\left(D_x\left(\frac{p}{a}\right)^4 + 2B\left(\frac{pq}{ab}\right)^2 + D_y\left(\frac{q}{b}\right)^4\right)} \tag{5}$$

where p and q are real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method (Pevzner et al. 2000).

3.   DYNAMIC RESPONSE

The orthotropic stiffened plate is assumed to be subjected to a normal blast loading $p(x,y,t)$ on the upper surface. The displacements can be expressed in the form of:

$$w_{mn}(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} X_m(x)Y_n(y)T_{mn}(t) \tag{6}$$

where $X_m(x)$, $Y_n(y)$ are Eigen functions, $T_{mn}(t)$ is a function of time, which must be determined through further analysis.

The differential equation for the function $T_{mn}(t)$ can be expressed as:

$$\ddot{T}_{mn}(t) + 2\gamma\omega_{mn}\dot{T}_{mn}(t) + \left(\omega_{mn}\right)^2 T_{mn}(t) = F(t) \tag{7}$$

The blast loading $P(t)$ which as a stepped triangular function can be expressed by:

$$P(t) = P_0\left(1 - \frac{t}{t_3}\right) \qquad \text{for } 0 \leq t \leq t_1 \tag{8}$$

$$P(t) = P_2\left(1 - \frac{t}{t_2}\right) \qquad \text{for } 0 \leq t \leq t_2 \tag{9}$$

$$P(t) = 0 \qquad \text{for } t_1 > t_2 \tag{10}$$

The orthotropic damped stiffened plate is assumed to be initially at rest; hence, the solution of equation (7) is obtained using the Duhamel convolution integral:

$$T_{mn}(t) = \left[\int_0^t \frac{P(\tau)\delta[x-x(t)]\delta[y-y(t)]}{\rho h Q_{mn}}\int_0^a X_m(x)dx\int_0^b Y_n(y)dy\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{1-\gamma^2}}\sin\left(\omega_{mn}(t-\tau)\right)d\tau\right] \tag{11}$$
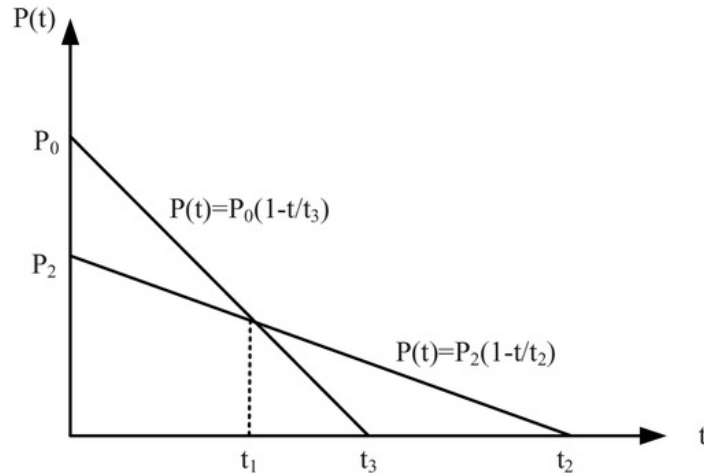
Figure 1: Stepped triangular loading

The general solution for the forced response deflection of the orthotropic stiffened plate to a stepped triangular load *P(t)* is given in integral form as follows:

$$w(x,y,t) = \sum_{m=1}^{m=\infty} X_{mn}(x) \sum_{n=1}^{n=\infty} Y_{mn}(y) \left[\int_0^t \frac{P(\tau)\delta[x - x(t)]\delta[y - y(t)]}{\rho h Q_{mn}} \int_0^a X_{mn}(x)dx \int_0^b Y_{mn}(y)dy \right.$$

$$\left. \frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{1-\gamma^2}} \sin\omega_{mn}(t-\tau)d\tau \right]$$

(12)

where is the damping ratio, $\delta[.]$ is the Diract delta function, Q_{mn} is the normalization factor of the Eigen vectors.

Once the response deflections have been obtained, the internal forces of the plate (moment and shear forces) can be computed, using derivatives of those deflections.

4.   **NUMERICAL RESULTS**

A reinforced concrete rectangular damped plate stiffened by rectangular stiffeners along the x axis is considered. The material is assumed to be orthotropic and linearly elastic. The data for the orthotropic plate and blast load are: a=8 m, b=4.75 m, h=0.12 m, E_c= 2.35 x 10^{10} N/m^2,   =0.2347,   = 2400 kg/m^3, t_1=1.55844 ms, t_2=40 ms, t_3=3 ms, P_0=300000 N/m^2, P_2=150000 N/m^2. The boundary conditions are fully fixed along the x and y edges. In the following discussion the position of the blast load is at $x=x_0$=1 m and $y=y_0$=1 m. The absolute maximum deflection at the mid plate due a stepped triangular load has been calculated by using 4 modes in the x direction (m=1,2,…,4) and 5 modes in the y direction (n=1,2,…,5). Two transcendental equations are used to obtain the values of p and q and the natural frequencies of the orthotropic plate for three models; orthotropic plate model 1 (without stiffener), model 2 (1 stiffener) and model 3 (2 stiffeners). The natural frequencies of the plate are shown in Table 1.

Table 1: The fundamental frequencies for the first 4 modes in x direction (m=1,2,…,4) and for the first 5 modes in y direction (n=1,2,…,5).

m	n	model 1 (without stiffener) ω_{mn} (rad/s)	model 2 (1 stiffener) ω_{mn} (rad/s)	model 3 (2 stiffeners) ω_{mn} (rad/s)
1	1	125.9377866	188.3313496	212.7882778
	2	321.1201405	497.4097793	565.2984477
	3	614.1143919	961.1598502	1094.178617
	4	1004.501306	1578.986048	1798.743282
	5	1492.38818	2351.101462	2604.493536
2	1	179.9294036	235.2811702	258.6256722
	2	371.9715048	544.07534	612.2010444
	3	664.5661617	1008.34844	1142.029541
	4	1055.022605	1626.67312	1847.305212
	5	1543.038698	2399.155375	2659.785338
3	1	275.3098224	323.1016134	344.5518271
	2	460.2572929	625.5612688	693.5077427
	3	750.4670756	1088.71936	1223.146606
	4	1140.203579	1707.066043	1928.919168
	5	1628.021834	2479.79656	2751.802832
4	1	408.7201683	451.1831828	470.9300733
	2	586.7716689	744.2564237	811.4820646
	3	873.0116244	1203.961159	1338.920235
	4	1260.94082	1821.242645	2044.413525
	5	1747.962181	2593.711779	2880.359691

Table 2: The absolute maximum dynamic deflection of a damped orthotropic stiffened plate subjected to a stepped triangular load for different values of damping ratio and stiffeners configuration.

Damping ratio	model 1 w_{max} (mm)	model 2 w_{max} (mm)	model 3 w_{max} (mm)
1%	14.45	10.17	9.51
3%	10.45	6.47	5.21
5%	7.87	4.41	3.4
10%	4.43	2.11	1.79

4.1   Effect of stiffeners configurations

The absolute maximum dynamic deflection has been computed for 3 different stiffeners configuration. The introduction of stiffeners decreases the mid-point displacement significantly; the mid-point displacement for model 1 (without stiffener) is 7.86 mm, while for models 2 and 3 are 4.41 mm and 3.40 mm respectively, all computed for the value of damping ratio of 5%. Thus, the configurations of stiffeners can have an important influence on the response of the stiffened orthotropic plates as shown in Table 2.

4.2   Effect of time duration

For model 1 with the value of damping ratio of 3%, 5% and 10% increasing the time duration of the step triangular load (t_3) by a factor of 5/3 has resulted in an increase in the mid-point displacement by a factor of 1.51, 1.50 and 1.48 respectively. For model 2 with the value of damping ratio of 3%, 5% and 10% increasing the time duration of the step triangular load (t_3) by a factor of 5/3 has resulted in an increase in the mid-point displacement by a factor of 1.54, 1.56 and 1.59 respectively. Therefore, the time duration of the blast loading plays an important role in determining the level of response of the orthotropic plate.
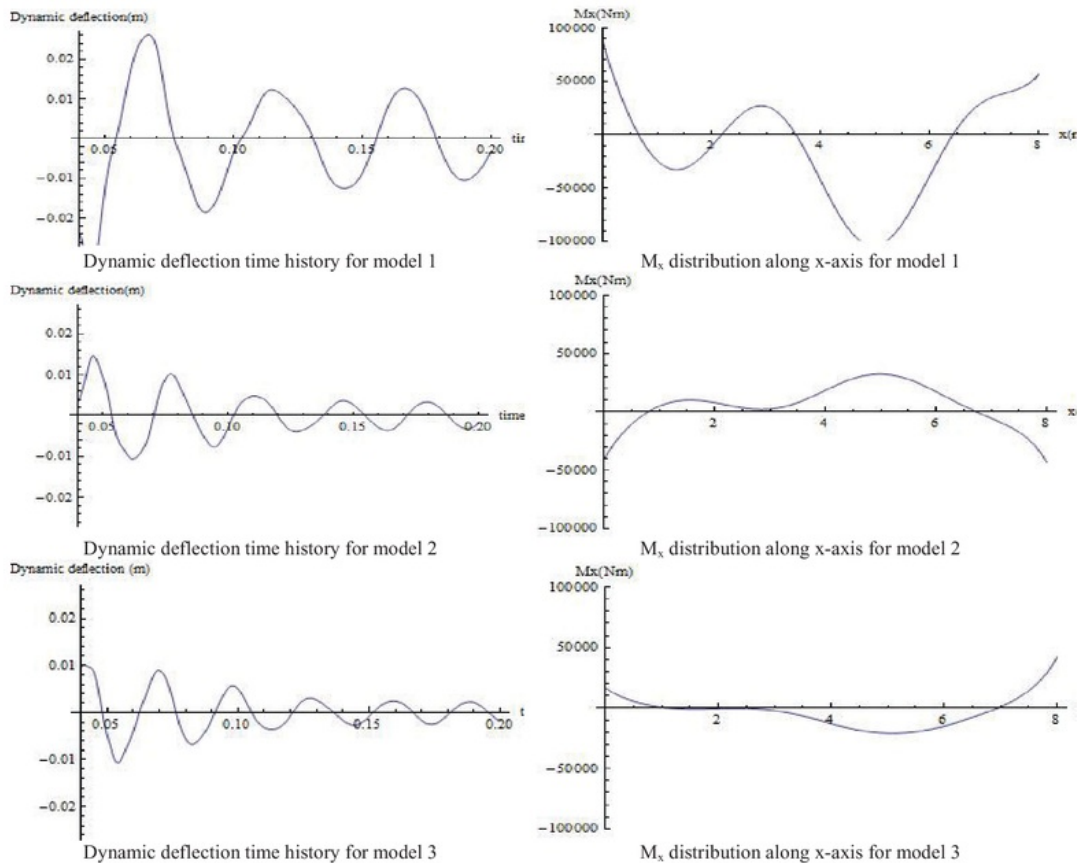
Dynamic deflection time history for model 1	M_x distribution along x-axis for model 1
Dynamic deflection time history for model 2	M_x distribution along x-axis for model 2
Dynamic deflection time history for model 3	M_x distribution along x-axis for model 3

Figure 2: Response of damped orthotropic plate for model 1, model 2 and model 3 subjected to a stepped triangular load.

4.3   Dynamic deflection time history

In Figure 2 the mid-point displacement time histories and the x-moment (M_x) distribution along the x-axes are shown for damping ratio equal to 5% and t_3=3 ms for model 1, model 2 and model 3. It can be seen, that the dynamic deflection of the mid-point and the x-moment distribution along x-axes of model 2 and model 3 are relatively smaller than that of model 1 due to the greater stiffeners effect. It can also be

seen, that the mid-point deflection are decaying with time due to the damping effect. The deformation of the stiffened orthotropic plate at different times is shown in Figure 3.

t=0.002 s   t=0.0025 s

t=0.003 s   t=0.0035 s

t=0.004 s   t=0.0045 s

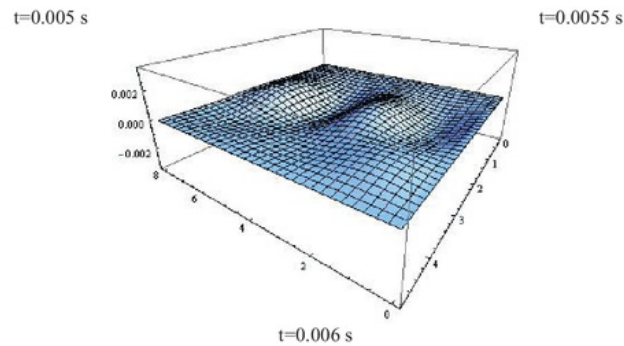t=0.005 s   t=0.0055 s

t=0.006 s
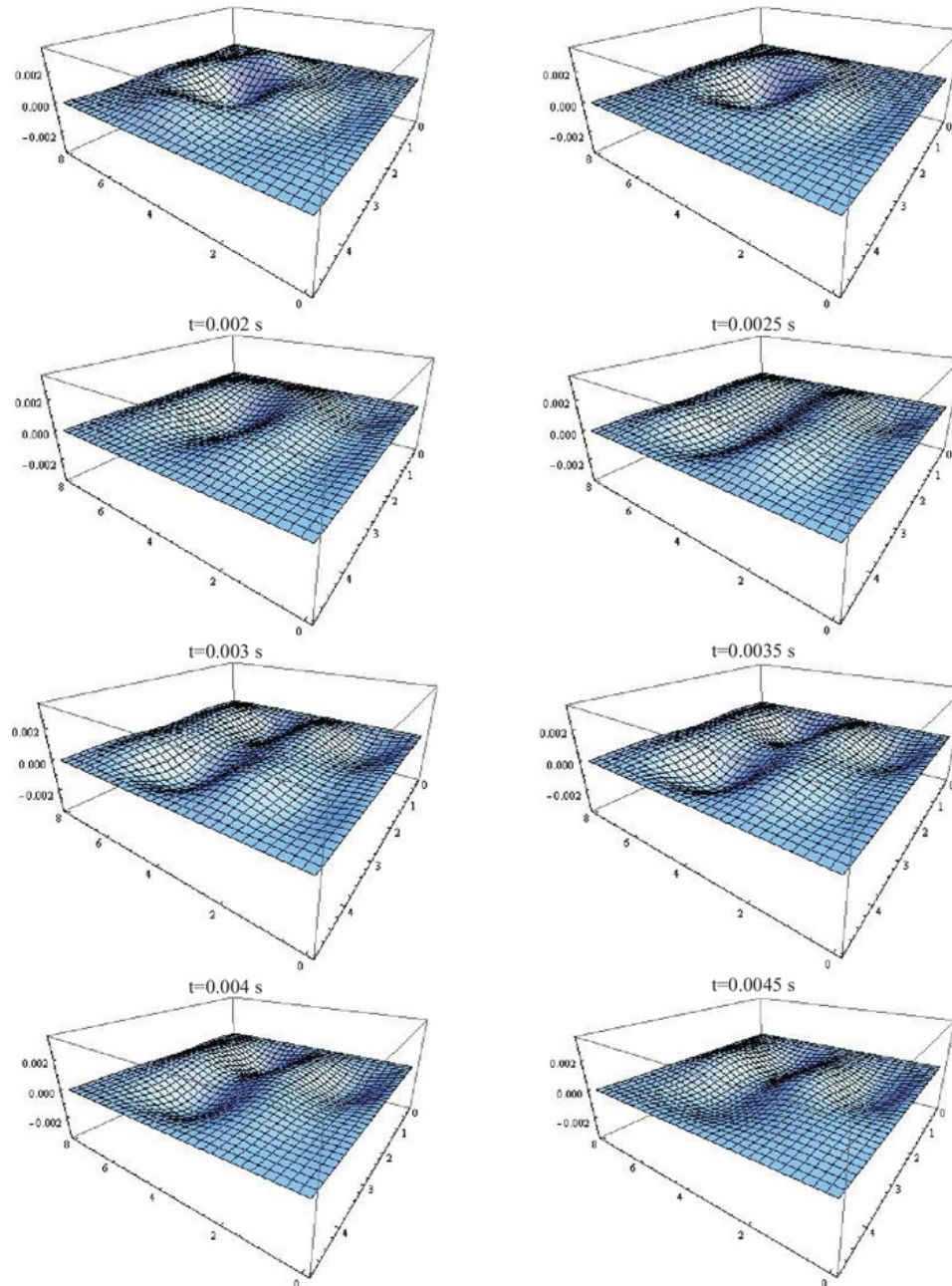
Figure 3: The deformation of the damped orthotropic plate with 2 stiffeners with the value of damping ratio 5% subjected to a stepped triangular load.

5.   CONCLUSIONS

From the dynamic analyses carried out to examine the behavior of the fully fixed stiffened and un-stiffened orthotropic plates under a step triangular blast load, the following conclusions can be drawn:

1. The effect of stiffeners configurations can be very important, since it affects drastically the overall behavior of the stiffened orthotropic plate.
2. The time duration of a stepped triangular blast load (t_3) is one of the most important parameters, since it has an influence on other responses, such as moment distribution and the maximum dynamic deflection of the system.
3. The inclusion of damping in calculating the dynamic response of the system will result in a much stiffer responses, especially for large values of t_3 resulting in lower mid-point deflection of the orthotropic plate.

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